

7.14 1)

$$\begin{aligned}\lim_{x \rightarrow +\infty} (\sqrt{4x+1} - \sqrt{x-2}) &= \lim_{x \rightarrow +\infty} \left(\sqrt{x \left(4 + \frac{1}{x} \right)} - \sqrt{x \left(1 - \frac{2}{x} \right)} \right) = \\ &= \lim_{x \rightarrow +\infty} \left(\sqrt{x} \sqrt{4 + \frac{1}{x}} - \sqrt{x} \sqrt{1 - \frac{2}{x}} \right) = \lim_{x \rightarrow +\infty} \sqrt{x} \left(\sqrt{4 + \frac{1}{x}} - \sqrt{1 - \frac{2}{x}} \right) = \\ &= \lim_{x \rightarrow +\infty} \sqrt{x} \left(\sqrt{4 + \frac{1}{x}} - \sqrt{1 - \frac{2}{x}} \right) = +\infty (\sqrt{4+0} - \sqrt{1-0}) = +\infty \cdot 1 = \boxed{+\infty}\end{aligned}$$

7.14 2)

$$\begin{aligned}\lim_{x \rightarrow +\infty} (\sqrt{x+1} + 2\sqrt{x-3}) &= \lim_{x \rightarrow +\infty} \left(\sqrt{x \left(1 + \frac{1}{x} \right)} + 2\sqrt{x \left(1 - \frac{3}{x} \right)} \right) \\ &= \lim_{x \rightarrow +\infty} \left(\sqrt{x} \sqrt{1 + \frac{1}{x}} + 2\sqrt{x} \sqrt{1 - \frac{3}{x}} \right) = \lim_{x \rightarrow +\infty} \sqrt{x} \left(\sqrt{1 + \frac{1}{x}} + 2\sqrt{1 - \frac{3}{x}} \right) = \\ &= (+\infty) (\sqrt{1+0} + 2\sqrt{1-0}) = \boxed{+\infty}\end{aligned}$$

7.14 3)

$$\begin{aligned}\lim_{x \rightarrow +\infty} \frac{\sqrt{x-1} + \sqrt{x+1}}{\sqrt{4x-2} - 3\sqrt{x-5}} &= \lim_{x \rightarrow +\infty} \frac{\sqrt{x \left(1 - \frac{1}{x} \right)} + \sqrt{x \left(1 + \frac{1}{x} \right)}}{\sqrt{x \left(4 - \frac{2}{x} \right)} - 3\sqrt{x \left(1 - \frac{5}{x} \right)}} = \\ &= \lim_{x \rightarrow +\infty} \frac{\sqrt{x \left(1 - \frac{1}{x} \right)} + \sqrt{x \left(1 + \frac{1}{x} \right)}}{\sqrt{x \left(4 - \frac{2}{x} \right)} - 3\sqrt{x \left(1 - \frac{5}{x} \right)}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x} \sqrt{1 - \frac{1}{x}} + \sqrt{x} \sqrt{1 + \frac{1}{x}}}{\sqrt{x} \sqrt{4 - \frac{2}{x}} - 3\sqrt{x} \sqrt{1 - \frac{5}{x}}} = \\ &= \lim_{x \rightarrow +\infty} \frac{\cancel{\sqrt{x}} \left(\sqrt{1 - \frac{1}{x}} + \sqrt{1 + \frac{1}{x}} \right)}{\cancel{\sqrt{x}} \left(\sqrt{4 - \frac{2}{x}} - 3\sqrt{1 - \frac{5}{x}} \right)} = \frac{\sqrt{1-0} + \sqrt{1+0}}{\sqrt{4-0} - 3\sqrt{1-0}} = \frac{1+1}{2-3 \cdot 1} = \boxed{-2}\end{aligned}$$