Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

7.13 1)

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2+3}}{2x+5}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(4+\frac{3}{x^2}\right)}}{x\left(2+\frac{5}{x}\right)}=\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{4+\frac{3}{x^2}}}{x\left(2+\frac{5}{x}\right)}=\lim_{x\to-\infty}\frac{|x|\sqrt{4+\frac{3}{x^2}}}{x\left(2+\frac{5}{x}\right)}=$$

$$\overset{|x|\overset{x<0}{=}-x}{=}\lim_{x\to-\infty}\frac{-\cancel{x}\sqrt{4+\frac{3}{x^2}}}{\cancel{x}\left(2+\frac{5}{x}\right)}=\frac{-\sqrt{4+0}}{2+0}=\frac{-2}{2}=\boxed{-1}$$

7.13 2)

$$\lim_{x\to+\infty}\frac{\sqrt{4x^2+3}}{2x+5}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(4+\frac{3}{x^2}\right)}}{x\left(2+\frac{5}{x}\right)}=\lim_{x\to+\infty}\frac{\sqrt{x^2}\sqrt{4+\frac{3}{x^2}}}{x\left(2+\frac{5}{x}\right)}=\lim_{x\to+\infty}\frac{|x|\sqrt{4+\frac{3}{x^2}}}{x\left(2+\frac{5}{x}\right)}=$$

$$\overset{|x|\overset{x>0}{=}x}{=}\lim_{x\to+\infty}\frac{\cancel{x}\sqrt{4+\frac{3}{x^2}}}{\cancel{x}\left(2+\frac{5}{x}\right)}=\frac{\sqrt{4+0}}{2+0}=\frac{2}{2}=\boxed{1}$$

7.13 3)

$$\lim_{x\to+\infty}\frac{\sqrt{x^2-5x+6}}{x+7}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(1-\frac{5}{x}+\frac{6}{x^2}\right)}}{x\left(1+\frac{7}{x}\right)}=\lim_{x\to+\infty}\frac{\sqrt{x^2}\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}}{x\left(1+\frac{7}{x}\right)}=$$

$$=\lim_{x\to+\infty}\frac{|x|\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}}{x\left(1+\frac{7}{x}\right)}=\overset{|x|\overset{x>0}{=}x}{=}\lim_{x\to+\infty}\frac{\cancel{x}\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}}{\cancel{x}\left(1+\frac{7}{x}\right)}=\frac{\sqrt{1-0+0}}{1+0}=\frac{1}{1}=\boxed{1}$$

7.13 4)

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+x+3}}{2x-3}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{3}{x^2}\right)}}{x\left(2-\frac{3}{x}\right)}=\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{1+\frac{1}{x}+\frac{3}{x^2}}}{x\left(2-\frac{3}{x}\right)}=$$

$$=\lim_{x\to-\infty}\frac{|x|\sqrt{1+\frac{1}{x}+\frac{3}{x^2}}}{x\left(2-\frac{3}{x}\right)}=\overset{|x|\overset{x<0}{=}-x}{=}\lim_{x\to-\infty}\frac{-\cancel{x}\sqrt{1+\frac{1}{x}+\frac{3}{x^2}}}{\cancel{x}\left(2-\frac{3}{x}\right)}=\frac{-\sqrt{1+0+0}}{2-0}=\frac{-1}{2}=\boxed{-\frac{1}{2}}$$

7.13 5)

$$\lim_{x\to+\infty}\frac{\sqrt{x^2-x+5}}{x}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(1-\frac{1}{x}+\frac{5}{x^2}\right)}}{x}=\lim_{x\to+\infty}\frac{\sqrt{x^2}\sqrt{1-\frac{1}{x}+\frac{5}{x^2}}}{x}=$$

$$= \lim_{x\to+\infty} \frac{|x|\sqrt{1-\frac{1}{x}+\frac{5}{x^2}}}{x} = \overset{|x|\overset{x>0}{=}x}{=} \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1-\frac{1}{x}+\frac{5}{x^2}}}{\cancel{x}} = \sqrt{1-0+0} = \boxed{1}$$

7.13 6)

$$\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to+\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x^2}}} =$$

$$\overset{|x|\overset{x>0}{=}x}{=} \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = \boxed{1}$$

7.13 7)

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x^2}}} =$$

$$\overset{|x|\overset{x<0}{=}-x}{=} \lim_{x\to-\infty} \frac{\cancel{x}}{-\cancel{x}\sqrt{1+\frac{1}{x^2}}} = \frac{1}{-\sqrt{1+0}} = \boxed{-1}$$

7.13 8)

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+1}-2x+3}{x+1} = \lim_{x\to+\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}-2x+3}{x+1} = \lim_{x\to+\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}-2x+3}{x+1} =$$

$$= \lim_{x\to+\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}-2x+3}{x+1} \overset{|x|\overset{x>0}{=}x}{=} \lim_{x\to+\infty} \frac{x\sqrt{1+\frac{1}{x^2}}-2x+3}{x+1} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(\sqrt{1+\frac{1}{x^2}}-2+\frac{3}{x}\right)}{\cancel{x}\left(1+\frac{1}{x}\right)} = \frac{\sqrt{1+0}-2+0}{1+0} = \boxed{-1}$$

7.13 9)

$$\lim_{x\to-\infty} \frac{3x-\sqrt{x^2+x+1}}{x-1} = \lim_{x\to-\infty} \frac{3x-\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}}{x-1} =$$

$$= \lim_{x\to-\infty} \frac{3x-\sqrt{x^2}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x-1} = \lim_{x\to-\infty} \frac{3x-|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x-1} \overset{|x|\overset{x<0}{=}-x}{=}$$

$$= \lim_{x\to-\infty} \frac{3x+x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x-1} = \lim_{x\to-\infty} \frac{\cancel{x}\left(3+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}{\cancel{x}\left(1-\frac{1}{x}\right)} = \frac{3+\sqrt{1+0+0}}{1-0} = \boxed{4}$$

7.13 10)

$$\lim_{x\to+\infty}\frac{\sqrt{x^2+1}-x}{x+1}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(1+\frac{1}{x}\right)}-x}{x+1}=\lim_{x\to+\infty}\frac{\sqrt{x^2}\sqrt{1+\frac{1}{x}}-x}{x+1}=$$

$$=\lim_{x\to+\infty}\frac{|x|\sqrt{1+\frac{1}{x}}-x}{x+1}\overset{|x|\overset{x>0}{=}x}{=}\lim_{x\to+\infty}\frac{x\sqrt{1+\frac{1}{x}}-x}{x+1}=\lim_{x\to+\infty}\frac{\cancel{x}\left(\sqrt{1+\frac{1}{x}}-1\right)}{\cancel{x}\left(1+\frac{1}{x}\right)}=\frac{\sqrt{1+0}-1}{1+0}=\boxed{0}$$

7.13 11)

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2+1}-x}{\sqrt{x^2+1}+3x}=\lim_{x\to-\infty}\frac{\sqrt{4x^2+1}-x}{\sqrt{x^2+1}+3x}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(4+\frac{1}{x^2}\right)}-x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+3x}=$$

$$=\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{4+\frac{1}{x^2}}-x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}+3x}=\lim_{x\to-\infty}\frac{|x|\sqrt{4+\frac{1}{x^2}}-x}{|x|\sqrt{1+\frac{1}{x^2}}+3x}\overset{|x|\overset{x<0}{=}-x}{=}\lim_{x\to-\infty}\frac{-x\sqrt{4+\frac{1}{x^2}}-x}{-x\sqrt{1+\frac{1}{x^2}}+3x}=$$

$$=\lim_{x\to-\infty}\frac{-x\sqrt{4+\frac{1}{x^2}}-x}{-x\sqrt{1+\frac{1}{x^2}}+3x}=\lim_{x\to-\infty}\frac{\cancel{x}\left(-\sqrt{4+\frac{1}{x^2}}-1\right)}{\cancel{x}\left(-\sqrt{1+\frac{1}{x^2}}+3\right)}=\frac{-\sqrt{4+0}-1}{-\sqrt{1+0}+3}=\boxed{-\frac{3}{2}}$$

7.13 12)

$$\lim_{x\to-\infty}\frac{2x+3-\sqrt{4x^2+5x+7}}{2x+7}=\lim_{x\to-\infty}\frac{2x+3-\sqrt{x^2\left(4+\frac{5}{x}+\frac{7}{x^2}\right)}}{x\left(2+\frac{7}{x}\right)}=$$

$$=\lim_{x\to-\infty}\frac{2x+3-\sqrt{x^2}\sqrt{4+\frac{5}{x}+\frac{7}{x^2}}}{x\left(2+\frac{7}{x}\right)}=\lim_{x\to-\infty}\frac{2x+3-|x|\sqrt{4+\frac{5}{x}+\frac{7}{x^2}}}{x\left(2+\frac{7}{x}\right)}\overset{|x|\overset{x<0}{=}-x}{=}$$

$$=\lim_{x\to-\infty}\frac{2x+3+x\sqrt{4+\frac{5}{x}+\frac{7}{x^2}}}{x\left(2+\frac{7}{x}\right)}=\lim_{x\to-\infty}\frac{\cancel{x}\left(2+\frac{3}{x}+\sqrt{4+\frac{5}{x}+\frac{7}{x^2}}\right)}{\cancel{x}\left(2+\frac{7}{x}\right)}$$

$$=\frac{2+0+\sqrt{4+0+0}}{2+0}=\boxed{2}$$