Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

7.12 1)

α) $$\lim_{x\to+\infty}\left(\sqrt{9x^2+x+3}-2x\right)=\lim_{x\to+\infty}\left[\sqrt{x^2\left(9+\frac{1}{x}+\frac{3}{x^2}\right)}-2x\right]=$$

$$=\lim_{x\to+\infty}\left[\sqrt{x^2}\sqrt{9+\frac{1}{x}+\frac{3}{x^2}}-2x\right]==\lim_{x\to+\infty}\left[|x|\sqrt{9+\frac{1}{x}+\frac{3}{x^2}}-2x\right]\overset{|x|\overset{x>0}{=}x}{=}$$

$$=\lim_{x\to+\infty}\left[x\sqrt{9+\frac{1}{x}+\frac{3}{x^2}}-2x\right]=\lim_{x\to+\infty}\left[x\left(\sqrt{9+\frac{1}{x}+\frac{3}{x^2}}-2\right)\right]=$$

$$=(+\infty)\left(\sqrt{9+0+0}-2\right)=(+\infty)(3-2)=\boxed{+\infty}$$

β) $$\lim_{x\to-\infty}\left(\sqrt{4x^2+x+6}+x\right)=\lim_{x\to-\infty}\left[\sqrt{x^2\left(4+\frac{1}{x}+\frac{6}{x^2}\right)}+x\right]=$$

$$=\lim_{x\to-\infty}\left[\sqrt{x^2}\sqrt{4+\frac{1}{x}+\frac{6}{x^2}}+x\right]=\lim_{x\to-\infty}\left[|x|\sqrt{4+\frac{1}{x}+\frac{6}{x^2}}+x\right]\overset{|x|\overset{x<0}{=}-x}{=}$$

$$=\lim_{x\to-\infty}\left[-x\sqrt{4+\frac{1}{x}+\frac{6}{x^2}}+x\right]=\lim_{x\to-\infty}\left[x\left(-\sqrt{4+\frac{1}{x}+\frac{6}{x^2}}+1\right)\right]=$$

$$=(-\infty)\left(-\sqrt{4+0+0}+1\right)=(-\infty)(-2+1)=\boxed{+\infty}$$

7.12 2)

$$\lim_{x\to-\infty}\left(\sqrt{2x^2+4x-3}-5x\right)=\lim_{x\to-\infty}\left[\sqrt{x^2\left(2+\frac{4}{x}-\frac{3}{x^2}\right)}-5x\right]=$$

$$=\lim_{x\to-\infty}\left[\sqrt{x^2}\sqrt{2+\frac{4}{x}-\frac{3}{x^2}}-5x\right]=\lim_{x\to-\infty}\left[|x|\sqrt{2+\frac{4}{x}-\frac{3}{x^2}}-5x\right]\overset{|x|\overset{x<0}{=}-x}{=}$$

$$=\lim_{x\to-\infty}\left[-x\sqrt{2+\frac{4}{x}-\frac{3}{x^2}}-5x\right]=\lim_{x\to-\infty}\left[x\left(-\sqrt{2+\frac{4}{x}-\frac{3}{x^2}}-5x\right)\right]=$$

$$=(-\infty)\left(-\sqrt{2+0-0}-5\right)=(-\infty)\left(-\sqrt{2}-5\right)=\boxed{+\infty}$$

7.12 3)

$$\lim_{x\to-\infty}\left(x-\sqrt{x^2+1}\right)=\lim_{x\to-\infty}\left(x-\sqrt{x^2\left(1+\frac{1}{x^2}\right)}\right)=$$

$$=\lim_{x\to-\infty}\left(x-\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}\right)=\lim_{x\to-\infty}\left(x-|x|\sqrt{1+\frac{1}{x^2}}\right)\overset{|x|\overset{x<0}{=}-x}{=}$$

$$=\lim_{x\to-\infty}\left(x-(-x)\sqrt{1+\frac{1}{x^2}}\right)=\lim_{x\to-\infty}\left(x+x\sqrt{1+\frac{1}{x^2}}\right)=$$

$$= \lim_{x\to-\infty} x\left(1+\sqrt{1+\frac{1}{x^2}}\right) = (-\infty)\left(1+\sqrt{1+0}\right) = \boxed{-\infty}$$

7.12 4)

$$\lim_{x\to+\infty}\left(\sqrt{x^2+3}-2x\right) = \lim_{x\to+\infty}\left[\sqrt{x^2\left(1+\frac{3}{x^2}\right)}-2x\right] =$$

$$= \lim_{x\to+\infty}\left[\sqrt{x^2}\sqrt{1+\frac{3}{x^2}}-2x\right] = \lim_{x\to+\infty}\left[|x|\sqrt{1+\frac{3}{x^2}}-2x\right] \overset{|x| \overset{x>0}{=} x}{=}$$

$$= \lim_{x\to+\infty}\left[x\sqrt{1+\frac{3}{x^2}}-2x\right] = \lim_{x\to+\infty}\left[x\left(\sqrt{1+\frac{3}{x^2}}-2\right)\right] =$$

$$= (+\infty)\left(\sqrt{1+0}-2\right) = (+\infty)(1-2) = \boxed{-\infty}$$

7.12 5)

$$\lim_{x\to+\infty}\left(\sqrt{x^2+4}-\sqrt{9x^2+3}\right) = \lim_{x\to+\infty}\left(\sqrt{x^2\left(1+\frac{4}{x^2}\right)}-\sqrt{x^2\left(9+\frac{3}{x^2}\right)}\right) =$$

$$= \lim_{x\to+\infty}\left(\sqrt{x^2}\sqrt{1+\frac{4}{x^2}}-\sqrt{x^2}\sqrt{9+\frac{3}{x^2}}\right) = \lim_{x\to+\infty}\left(|x|\sqrt{1+\frac{4}{x^2}}-|x|\sqrt{9+\frac{3}{x^2}}\right) \overset{|x| \overset{x>0}{=} x}{=}$$

$$= \lim_{x\to+\infty}\left(x\sqrt{1+\frac{4}{x^2}}-x\sqrt{9+\frac{3}{x^2}}\right) = \lim_{x\to+\infty} x\left(\sqrt{1+\frac{4}{x^2}}-\sqrt{9+\frac{3}{x^2}}\right) =$$

$$= (+\infty)\left(\sqrt{1+0}-\sqrt{9+0}\right) = (+\infty)(-2) = \boxed{-\infty}$$

7.12 6)

$$\lim_{x\to-\infty}\left(\sqrt{4x^2+x}-\sqrt{9x^2+1}\right) = \lim_{x\to-\infty}\left(\sqrt{x^2\left(4+\frac{1}{x}\right)}-\sqrt{x^2\left(9+\frac{1}{x^2}\right)}\right) =$$

$$= \lim_{x\to-\infty}\left(\sqrt{x^2}\sqrt{4+\frac{1}{x}}-\sqrt{x^2}\sqrt{9+\frac{1}{x^2}}\right) = \lim_{x\to-\infty}\left(|x|\sqrt{4+\frac{1}{x}}-|x|\sqrt{9+\frac{1}{x^2}}\right) \overset{|x| \overset{x<0}{=} -x}{=}$$

$$= \lim_{x\to-\infty}\left(-x\sqrt{4+\frac{1}{x}}-(-x)\sqrt{9+\frac{1}{x^2}}\right) = \lim_{x\to-\infty}\left(-x\sqrt{4+\frac{1}{x}}+x\sqrt{9+\frac{1}{x^2}}\right) =$$

$$= \lim_{x\to-\infty} x\left(-\sqrt{4+\frac{1}{x}}+\sqrt{9+\frac{1}{x^2}}\right) = (-\infty)\left(-\sqrt{4+0}+\sqrt{9+0}\right) = \boxed{-\infty}$$

7.12 7)

$$\lim_{x\to-\infty}\left(\sqrt{4x^2+3x-4}-\sqrt{x^2+x}\right) = \lim_{x\to-\infty}\left(\sqrt{x^2\left(4+\frac{3}{x}-\frac{4}{x^2}\right)}-\sqrt{x^2\left(1+\frac{1}{x}\right)}\right) =$$

$$= \lim_{x\to-\infty}\left(\sqrt{x^2}\sqrt{4+\frac{3}{x}-\frac{4}{x^2}}-\sqrt{x^2}\sqrt{1+\frac{1}{x}}\right) = \lim_{x\to-\infty}\left(|x|\sqrt{4+\frac{3}{x}-\frac{4}{x^2}}-|x|\sqrt{1+\frac{1}{x}}\right) \overset{|x| \overset{x<0}{=} -x}{=}$$

$$= \lim_{x\to-\infty}\left(-x\sqrt{4+\frac{3}{x}-\frac{4}{x^2}}+x\sqrt{1+\frac{1}{x}}\right) = \lim_{x\to-\infty} x\left(-\sqrt{4+\frac{3}{x}-\frac{4}{x^2}}+\sqrt{1+\frac{1}{x}}\right) =$$

$$= (-\infty)\left(-\sqrt{4+0-0}+\sqrt{1+0}\right) = (-\infty)(-2+1) = \boxed{+\infty}$$