

6.9 1)

Θέτουμε $h(x) = \frac{f(x)}{g(x)}$ οπότε προφανώς $\lim_{x \rightarrow x_0} h(x) = -\infty$ (1)

Ακόμη

$$h(x) = \frac{f(x)}{g(x)} \Rightarrow g(x) = \frac{f(x)}{h(x)} \Rightarrow \lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} \frac{f(x)}{h(x)} \stackrel{(1)}{\Rightarrow} \boxed{\lim_{x \rightarrow x_0} g(x) = \frac{\alpha}{-\infty} = 0}$$

6.9 2)

Θέτουμε $h(x) = \frac{f(x)}{g(x)}$ οπότε προφανώς $\lim_{x \rightarrow x_0} h(x) = +\infty$ (1)

Ακόμη

$$h(x) = \frac{f(x)}{g(x)} \Rightarrow g(x) = \frac{f(x)}{h(x)} \Rightarrow \lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} \frac{f(x)}{h(x)} \stackrel{(1)}{\Rightarrow} \boxed{\lim_{x \rightarrow x_0} g(x) = \frac{\alpha}{+\infty} = 0}$$

6.9 3)

Θέτουμε $h(x) = \frac{1+|x|}{f(x)}$ οπότε προφανώς $\lim_{x \rightarrow 2} h(x) = -\infty$ (1)

Ακόμη

$$h(x) = \frac{1+|x|}{f(x)} \Rightarrow f(x) = \frac{1+|x|}{h(x)} \Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{1+|x|}{h(x)} \stackrel{(1)}{\Rightarrow} \boxed{\lim_{x \rightarrow 2} f(x) = \frac{3}{-\infty} = 0}$$

6.9 4)

Έστω $\lim_{x \rightarrow 0} f(x) = \alpha$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^4} = -\infty \Rightarrow \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} \frac{1}{x^4} = -\infty \Rightarrow \alpha \cdot (+\infty) = -\infty$$

Οπότε επειδή γνωρίζουμε ότι $\alpha \neq 0$ υποχρεωτικά θα είναι $\alpha < 0$ ή $\alpha = -\infty$