Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

6.6 1)

$$\boxed{\lim_{x\to 2^+}\frac{x-1}{\sqrt{x+2}-2}}=\lim_{x\to 2^+}\frac{(x-1)\left(\sqrt{x+2}+2\right)}{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}=\lim_{x\to 2^+}\frac{(x-1)\left(\sqrt{x+2}+2\right)}{\sqrt{x+2}^2-2^2}=$$

$$=\lim_{x\to 2^+}\frac{(x-1)\left(\sqrt{x+2}+2\right)}{x+2-4}=\lim_{x\to 2^+}\frac{(x-1)\left(\sqrt{x+2}+2\right)}{x-2}=\lim_{x\to 2^+}(x-1)\left(\sqrt{x+2}+2\right)\cdot\lim_{x\to 2^+}\frac{1}{x-2}=$$

$$=(2-1)\left(\sqrt{2+2}+2\right)\cdot(+\infty)=4\cdot(+\infty)=\boxed{+\infty}$$

6.6 2)

$$\lim_{x\to 1^-}\frac{5x}{\sqrt{3x-2}-1}=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{\left(\sqrt{3x-2}-1\right)\left(\sqrt{3x-2}+1\right)}=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{\sqrt{3x-2}^2-1^2}=$$

$$=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{3x-2-1}=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{3x-3}=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{3(x-1)}=$$

$$=\lim_{x\to 1^-}\frac{5x\left(\sqrt{3x-2}+1\right)}{3}\cdot\lim_{x\to 1^-}\frac{1}{x-1}=\frac{5\cdot 1\left(\sqrt{3\cdot 1-2}+1\right)}{3}\cdot(-\infty)=\frac{10}{3}\cdot(-\infty)=\boxed{-\infty}$$

6.6 3)

$$\lim_{x\to -1}\frac{3x+5}{\sqrt{5x+6}-1}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{5x+6}+1}}{=}\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{\left(\sqrt{5x+6}-1\right)\left(\sqrt{5x+6}+1\right)}=$$

$$=\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{\sqrt{5x+6}^2-1^2}=\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{5x+6-1}=$$

$$=\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{5x+5}=\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{5(x+1)}=$$

$$=\lim_{x\to -1}\frac{(3x+5)\left(\sqrt{5x+6}+1\right)}{5}\cdot\lim_{x\to -1}\frac{1}{x+1}=\frac{[3(-1)+5]\left(\sqrt{5(-1)+6}+1\right)}{5}\cdot\lim_{x\to -1}\frac{1}{x+1}=$$

$$=\frac{4}{5}\cdot\lim_{x\to -1}\frac{1}{x+1}=\left.\begin{array}{l}\overset{\text{Αν } x>-1}{=}\frac{4}{5}\cdot\lim_{x\to -1^+}\frac{1}{x+1}=+\infty\\ \overset{\text{Αν } x<-1}{=}\frac{4}{5}\cdot\lim_{x\to -1^-}\frac{1}{x+1}=-\infty\end{array}\right\}\Rightarrow\lim_{x\to -1}\frac{3x+5}{\sqrt{5x+6}-1}\ \text{δεν υπάρχει}$$

6.6 4)

$$\lim_{x\to 0^+}\frac{x^2-3}{\sqrt{1+x}-\sqrt{1-x}}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{1+x}+\sqrt{1-x}}}{=}\lim_{x\to 0^+}\frac{(x^2-3)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+x}+\sqrt{1-x}\right)}=$$

$$=\lim_{x\to 0^+}\frac{(x^2-3)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\sqrt{1+x}^2-\sqrt{1-x}^2}=\lim_{x\to 0^+}\frac{(x^2-3)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{1+x-1+x}=$$

$$= \lim_{x \to 0^+} \frac{(x^2-3)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2x} = \lim_{x \to 0^+} \frac{(x^2-3)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2} \cdot \lim_{x \to 0^+} \frac{1}{x} =$$

$$= \frac{(0^2-3)\left(\sqrt{1+0}+\sqrt{1-0}\right)}{2} \cdot (+\infty) = (-3)\cdot(+\infty) = \boxed{-\infty}$$

6.6 5)

$$\lim_{x \to -2^-} \frac{2x+5}{\sqrt{x^2+5}-3} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } \sqrt{x^2+5}+3}}{=} \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{\left(\sqrt{x^2+5}-3\right)\left(\sqrt{x^2+5}+3\right)} =$$

$$= \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{\sqrt{x^2+5}^2-3^2} = \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{x^2+5-9} =$$

$$= \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{x^2-4} = \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{(x-2)(x+2)} =$$

$$= \lim_{x \to -2^-} \frac{(2x+5)\left(\sqrt{x^2+5}+3\right)}{x-2} \cdot \lim_{x \to -2^-} \frac{1}{x+2} = \frac{[2(-2)+5]\left(\sqrt{(-2)^2+5}+3\right)}{-2-2} \cdot (-\infty) =$$

$$= \frac{[2(-2)+5]\left(\sqrt{(-2)^2+5}+3\right)}{-2-2} \cdot (-\infty) = -\frac{6}{4} \cdot (-\infty) = \boxed{+\infty}$$

6.6 6)

$$\lim_{x \to -1^+} \frac{x+3}{3x+\sqrt{4x^2+5}} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 3x-\sqrt{4x^2+5}}}{=} \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{\left(3x+\sqrt{4x^2+5}\right)\left(3x-\sqrt{4x^2+5}\right)} =$$

$$= \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{(3x)^2-\sqrt{4x^2+5}^2} = \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{9x^2-4x^2-5} =$$

$$= \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{5x^2-5} = \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{5(x^2-1)} =$$

$$= \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{5(x-1)(x+1)} = \lim_{x \to -1^+} \frac{(x+3)\left(3x-\sqrt{4x^2+5}\right)}{5(x-1)} \cdot \lim_{x \to -1^+} \frac{1}{x+1} =$$

$$= \frac{(-1+3)\left[3(-1)-\sqrt{4(-1)^2+5}\right]}{5(-1-1)} \cdot (+\infty) = \frac{6}{5} \cdot (+\infty) = \boxed{+\infty}$$

6.6 7)

$$\lim_{x \to 1} \frac{\sqrt{x}-4}{x^3-3x+2} \overset{\substack{x^3-3x+2\text{: ρίζα το } x=1 \\ \text{παραγοντοποίηση με Horner}}}{=} \lim_{x \to 1} \frac{\sqrt{x}-4}{(x-1)^2(x+2)} = \lim_{x \to 1} \frac{\sqrt{x}-4}{x+2} \cdot \lim_{x \to 1} \frac{1}{(x-1)^2} =$$

$$= \frac{\sqrt{1}-4}{1+2} \cdot (+\infty) = -1 \cdot (+\infty) = \boxed{-\infty}$$