6.34

Θέτουμε $h(x)=\dfrac{f(x)\eta\mu(\pi x)}{\sqrt{x}-1}$ οπότε προφανώς $\lim\limits_{x\to 1}h(x)=+\infty$ (1)

Ακόμη

$$h(x)=\frac{f(x)\eta\mu(\pi x)}{\sqrt{x}-1}\Rightarrow f(x)=\frac{h(x)\left(\sqrt{x}-1\right)}{\eta\mu(\pi x)}\Rightarrow \lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{h(x)\left(\sqrt{x}-1\right)}{\eta\mu(\pi x)}\Rightarrow$$

$$\Rightarrow \lim_{x\to 1}f(x)=\lim_{x\to 1}h(x)\cdot\lim_{x\to 1}\frac{\sqrt{x}-1}{\eta\mu(\pi x)}\quad (2)$$

Όμως

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\eta\mu(\pi x)}\overset{\substack{\text{θέτουμε } y=x-1\Rightarrow x=y+1\\ \text{όταν } x\to 1,\ \text{τότε } y\to 0}}{=}\lim_{y\to 0}\frac{\sqrt{y+1}-1}{\eta\mu\left[\pi(y+1)\right]}=\lim_{y\to 0}\frac{\sqrt{y+1}-1}{\eta\mu(\pi y+\pi)}\overset{\eta\mu(\pi y+\pi)=-\eta\mu(\pi y)}{=}$$

$$=\lim_{y\to 0}\frac{\sqrt{y+1}-1}{-\eta\mu(\pi y)}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με την συζυγή}\\ \text{παράσταση}}}{=}\lim_{y\to 0}\frac{\left(\sqrt{y+1}-1\right)\left(\sqrt{y+1}+1\right)}{-\eta\mu(\pi y)\left(\sqrt{y+1}+1\right)}=$$

$$=\lim_{y\to 0}\frac{y}{-\eta\mu(\pi y)\left(\sqrt{y+1}+1\right)}=\lim_{y\to 0}\frac{1}{-\left(\sqrt{y+1}+1\right)}\cdot\lim_{y\to 0}\frac{y}{\eta\mu(\pi y)}=$$

$$=-\frac{1}{2}\cdot\lim_{y\to 0}\frac{\pi y}{\pi\eta\mu(\pi y)}\overset{\lim\limits_{y\to 0}\frac{\pi y}{\eta\mu(\pi y)}=1}{=}-\frac{1}{2\pi}$$

Δηλαδή $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{\eta\mu(\pi x)}=-\dfrac{1}{2\pi}$ (3)

Επομένως

$$(2)\overset{(1),(3)}{\Rightarrow}\boxed{\lim_{x\to 1}f(x)}=(+\infty)\left(-\frac{1}{2\pi}\right)=\boxed{-\infty}$$