

6.3 1)

$$\alpha) \lim_{x \rightarrow -1^+} \frac{x-2}{x+1} = \lim_{x \rightarrow -1^+} (x-2) \cdot \lim_{x \rightarrow -1^+} \frac{1}{x+1} = (-1-2) \cdot (+\infty) = \boxed{-\infty}$$

$$\beta) \lim_{x \rightarrow 3} \frac{x+1}{x-3} = \lim_{x \rightarrow 3} (x+1) \cdot \lim_{x \rightarrow 3} \frac{1}{x-3} = 4 \cdot \lim_{x \rightarrow 3} \frac{1}{x-3} =$$

$$= \left. \begin{array}{l} \nearrow \text{Av } x > 3 \\ 4 \cdot \lim_{x \rightarrow 3^+} \frac{1}{x-3} = 4 \cdot (+\infty) = +\infty \\ \searrow \text{Av } x < 3 \\ 4 \cdot \lim_{x \rightarrow 3^-} \frac{1}{x-3} = 4 \cdot (-\infty) = -\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 3} \frac{x+1}{x-3} \text{ δεν υπάρχει}$$

$$\gamma) \lim_{x \rightarrow 3} \frac{2-x^2}{(x+1)(x-3)^2} = \lim_{x \rightarrow 3} \frac{2-x^2}{x+1} \cdot \lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = \frac{2-3^2}{3+1} \cdot (+\infty) = -\frac{7}{4} \cdot (+\infty) = \boxed{-\infty}$$

$$\delta) \lim_{x \rightarrow -7} \frac{4-2x}{x(x+7)^3} = \lim_{x \rightarrow -7} \frac{4-2x}{x} \cdot \lim_{x \rightarrow -7} \frac{1}{(x+7)^3} = -\frac{18}{7} \cdot \lim_{x \rightarrow -7} \frac{1}{(x+7)^3}$$

$$= \left. \begin{array}{l} \nearrow \text{Av } x > 3 \\ 4 \cdot \lim_{x \rightarrow 3^+} \frac{1}{x-3} = 4 \cdot (+\infty) = +\infty \\ \searrow \text{Av } x < 3 \\ 4 \cdot \lim_{x \rightarrow 3^-} \frac{1}{x-3} = 4 \cdot (-\infty) = -\infty \end{array} \right\} \text{Άρα το } \lim_{x \rightarrow 3} \frac{x+1}{x-3} \text{ δεν υπάρχει}$$

6.3 2)

$$\lim_{x \rightarrow 0^+} \frac{3x+5}{x} = \lim_{x \rightarrow 0^+} (3x+5) \cdot \lim_{x \rightarrow 0^+} \frac{1}{x} = (3 \cdot 0 + 5) \cdot (+\infty) = \boxed{+\infty}$$

6.3 3)

$$\lim_{x \rightarrow 0} \frac{2x-1}{x^2} = \lim_{x \rightarrow 0} (2x-1) \cdot \lim_{x \rightarrow 0} \frac{1}{x^2} = (2 \cdot 0 - 1) \cdot (+\infty) = \boxed{-\infty}$$

6.3 4)

$$\lim_{x \rightarrow 0} \frac{x^2+3}{x} = \lim_{x \rightarrow 0} (x^2+3) \cdot \lim_{x \rightarrow 0} \frac{1}{x} = 3 \cdot \lim_{x \rightarrow 0} \frac{1}{x} = \left. \begin{array}{l} \nearrow \text{Av } x > 0 \\ 3 \cdot \lim_{x \rightarrow 0^+} \frac{1}{x} = 3 \cdot (+\infty) = +\infty \\ \searrow \text{Av } x < 0 \\ 3 \cdot \lim_{x \rightarrow 0^-} \frac{1}{x} = 3 \cdot (-\infty) = -\infty \end{array} \right\}$$

$$\text{Άρα το } \lim_{x \rightarrow 0} \frac{x^2+3}{x} \text{ δεν υπάρχει}$$

6.3 5)

$$\lim_{x \rightarrow 0} \frac{x^3-2}{|x|} = \lim_{x \rightarrow 0} (x^3-2) \cdot \lim_{x \rightarrow 0} \frac{1}{|x|} = (0^3-2) \cdot (+\infty) = \boxed{-\infty}$$

6.3 6)

$$\lim_{x \rightarrow 1} \frac{2x-7}{(x-1)^3} = \lim_{x \rightarrow 1} (2x-7) \cdot \lim_{x \rightarrow 1} \frac{1}{(x-1)^3} = -5 \cdot \lim_{x \rightarrow 1} \frac{1}{(x-1)^3} =$$

$$= \begin{cases} \nearrow_{\text{Av } x > 1} -5 \cdot \lim_{x \rightarrow 1^+} \frac{1}{(x-1)^3} = -5 \cdot (+\infty) = -\infty \\ \searrow_{\text{Av } x < 1} -5 \cdot \lim_{x \rightarrow 1^-} \frac{1}{(x-1)^3} = -5 \cdot (-\infty) = +\infty \end{cases}$$

Άρα το $\lim_{x \rightarrow 1} \frac{2x-7}{(x-1)^3}$ δεν υπάρχει

6.3 7)

$$\lim_{x \rightarrow 2^+} \frac{3x+1}{2-x} = \lim_{x \rightarrow 2^+} (3x+1) \cdot \lim_{x \rightarrow 2^+} \frac{1}{2-x} = (3 \cdot 2 + 1)(-\infty) = \boxed{-\infty}$$

6.3 8)

$$\lim_{x \rightarrow 6} \frac{2x}{(x-6)^4} = \lim_{x \rightarrow 6} 2x \cdot \lim_{x \rightarrow 6} \frac{1}{(x-6)^4} = 12 \cdot (+\infty) = \boxed{+\infty}$$

6.3 9)

$$\lim_{x \rightarrow 5} \frac{x^2-10}{(x-5)^2(2x-4)} = \lim_{x \rightarrow 5} \frac{x^2-10}{2x-4} \cdot \lim_{x \rightarrow 5} \frac{1}{(x-5)^2} = \frac{5^2-10}{2 \cdot 5-4} \cdot (+\infty) = \frac{15}{11} \cdot (+\infty) = \boxed{+\infty}$$

6.3 10)

$$\lim_{x \rightarrow -3} \frac{x-2}{(x+3)^3(4x-1)} = \lim_{x \rightarrow -3} \frac{x-2}{4x-1} \cdot \lim_{x \rightarrow -3} \frac{1}{(x+3)^3} = \frac{-5}{-13} \cdot \lim_{x \rightarrow -3} \frac{1}{(x+3)^3} = \frac{5}{13} \cdot \lim_{x \rightarrow -3} \frac{1}{(x+3)^3}$$

$$= \begin{cases} \nearrow_{\text{Av } x > -3} \frac{5}{13} \cdot \lim_{x \rightarrow -3^+} \frac{1}{(x+3)^3} = \frac{5}{13} \cdot (+\infty) = +\infty \\ \searrow_{\text{Av } x < -3} \frac{5}{13} \cdot \lim_{x \rightarrow -3^-} \frac{1}{(x+3)^3} = \frac{5}{13} \cdot (-\infty) = -\infty \end{cases} \Rightarrow \lim_{x \rightarrow -3} \frac{x-2}{(x+3)^3(4x-1)} \text{ δεν υπάρχει}$$

6.3 11)

$$\lim_{x \rightarrow -1^+} \frac{7-2x}{(2x+1)(x+1)} = \lim_{x \rightarrow -1^+} \frac{7-2x}{2x+1} \cdot \lim_{x \rightarrow -1^+} \frac{1}{x+1} = \frac{7-2(-1)}{2(-1)+1} \cdot (+\infty) = \frac{9}{-1} \cdot (+\infty) = \boxed{-\infty}$$

6.3 12)

$$\lim_{x \rightarrow 2} \frac{3x^2-17}{(x-2)^4(x-4)} = \lim_{x \rightarrow 2} \frac{3x^2-17}{x-4} \cdot \lim_{x \rightarrow 2} \frac{1}{(x-2)^4} = \frac{3 \cdot 2^2-17}{2-4} \cdot (+\infty) = \frac{-5}{-2} \cdot (+\infty) = \boxed{+\infty}$$

6.3 13)

$$\lim_{x \rightarrow 8} \frac{x-2}{(1-x)(x+8)^5}$$

$$\lim_{x \rightarrow 8} \frac{x-2}{(1-x)(x+8)^5} = \lim_{x \rightarrow 8} \frac{x-2}{1-x} \cdot \lim_{x \rightarrow 8} \frac{1}{(x+8)^5} = \frac{-10}{9} \cdot \lim_{x \rightarrow 8} \frac{1}{(x+8)^5} =$$

$$= \left. \begin{array}{l} \xrightarrow{\text{Av } x > -8} \frac{-10}{9} \cdot \lim_{x \rightarrow -8^+} \frac{1}{(x+8)^5} = \frac{-10}{9} \cdot (+\infty) = -\infty \\ \xrightarrow{\text{Av } x < -8} \frac{-10}{9} \cdot \lim_{x \rightarrow -8^-} \frac{1}{(x+8)^5} = \frac{-10}{9} \cdot (-\infty) = +\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow -8} \frac{x-2}{(1-x)(x+8)^5} \text{ δεν υπάρχει}$$