

ΓΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

6.26

$$\text{Θέτουμε } h(x) = \frac{f(x)-x}{x-1} \text{ οπότε προφανώς } \lim_{x \rightarrow 1} h(x) = 2 \quad (1)$$

Οπότε

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{f(x)+1}{[f(x)-1]^2} &\stackrel{(2)}{=} \lim_{x \rightarrow 1} \frac{(x-1)h(x)+x+1}{[(x-1)h(x)+x-1]^2} = \lim_{x \rightarrow 1} \frac{(x-1)h(x)+x+1}{[(x-1)(h(x)+1)]^2} = \\ &= \lim_{x \rightarrow 1} \frac{(x-1)h(x)+x+1}{(x-1)^2(h(x)+1)^2} = \lim_{x \rightarrow 1} \frac{(x-1)h(x)+x+1}{(h(x)+1)^2} \cdot \lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = \\ &= \frac{(1-1) \cdot 2 + 1 + 1}{(2+1)^2} \cdot (+\infty) = \frac{2}{9} \cdot (+\infty) = \boxed{+\infty} \end{aligned}$$