

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

6.11 1)

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{2}{x\sqrt{x} - 2x - 4\sqrt{x} + 8} & \stackrel{\substack{\text{θέτουμε } y=\sqrt{x} \\ \text{όταν } x \rightarrow 4, y \rightarrow 2}}{=} \lim_{y \rightarrow 2} \frac{2}{y^3 - 2y^2 - 4y + 8} \stackrel{\substack{y^3 - 2y^2 - 4y + 8: \text{ ρίζα το } y=2 \\ \text{παραγοντοποίηση με Horner}}{=} \\ &= \lim_{y \rightarrow 2} \frac{2}{(y-2)^2(y+2)} = \lim_{y \rightarrow 2} \frac{2}{(y-2)^2(y+2)} = \lim_{y \rightarrow 2} \frac{2}{(y+2)} \cdot \lim_{y \rightarrow 2} \frac{1}{(y-2)^2} = \\ &= \frac{2}{4} \cdot (+\infty) = \boxed{+\infty} \end{aligned}$$

6.11 2)

$$\begin{aligned} \lim_{x \rightarrow 1^+} \frac{x-4}{x-3\sqrt{x}+2} & \stackrel{\text{θέτουμε } y=\sqrt{x}}{=} \lim_{y \rightarrow 1^+} \frac{y^2-4}{y^2-3y+2} \stackrel{y^2-3y+2=(y-1)(y-2)}{=} \lim_{y \rightarrow 1^+} \frac{y^2-4}{(y-1)(y-2)} = \\ &= \lim_{y \rightarrow 1^+} \frac{y^2-4}{y-2} \cdot \lim_{y \rightarrow 1^+} \frac{1}{y-2} = \frac{1^2-4}{1-2} \cdot (+\infty) = \frac{3}{2} \cdot (+\infty) = \boxed{+\infty} \end{aligned}$$

6.11 3)

$$\begin{aligned} \lim_{x \rightarrow 4^+} \frac{3x-1}{x-5\sqrt{x}+6} & \stackrel{\text{θέτουμε } y=\sqrt{x}}{=} \lim_{y \rightarrow 2^+} \frac{3y^2-1}{y^2-5y+6} \stackrel{y^2-5y+6=(y-2)(y-3)}{=} \lim_{y \rightarrow 2^+} \frac{3y^2-1}{(y-2)(y-3)} = \\ &= \lim_{y \rightarrow 2^+} \frac{3y^2-1}{y-3} \cdot \lim_{y \rightarrow 2^+} \frac{1}{y-2} = \frac{3 \cdot 2^2 - 1}{2-3} \cdot (+\infty) = -7 \cdot (+\infty) = \boxed{-\infty} \end{aligned}$$

6.11 4)

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{5x+2}{x\sqrt{x}-x-\sqrt{x}+1} & \stackrel{\substack{\text{θέτουμε } y=\sqrt{x} \\ \text{όταν } x \rightarrow 1, y \rightarrow 1}}{=} \lim_{y \rightarrow 1} \frac{5y^2+2}{y^3-y^2-y+1} \stackrel{\substack{y^3-y^2-y+1: \text{ ρίζα το } y=1 \\ \text{παραγοντοποίηση με Horner}}{=} \Rightarrow \\ &= \lim_{y \rightarrow 1} \frac{5y^2+2}{(y-1)^2(y+1)} = \lim_{y \rightarrow 1} \frac{5y^2+2}{y+1} \cdot \lim_{y \rightarrow 1} \frac{1}{(y-1)^2} = \frac{7}{2} \cdot (+\infty) = \boxed{+\infty} \end{aligned}$$

6.11 5)

$$\begin{aligned} \lim_{x \rightarrow 4^-} \frac{x-7}{x^2-3\sqrt{x}-10} & \stackrel{\substack{\text{θέτουμε } y=\sqrt{x} \\ \text{όταν } x \rightarrow 4^-, y \rightarrow 2^-}}{=} \lim_{y \rightarrow 2^-} \frac{y^2-7}{y^4-3y-10} \stackrel{\substack{y^4-3y-10: \text{ ρίζα το } y=2 \\ \text{παραγοντοποίηση με Horner}}{=} \Rightarrow \\ &= \lim_{y \rightarrow 2^-} \frac{y^2-7}{(y-2)(y^3+2y^2+4y+5)} = \lim_{y \rightarrow 2^-} \frac{y^2-7}{y^3+2y^2+4y+5} \cdot \lim_{y \rightarrow 2^-} \frac{1}{y-2} = \frac{-3}{29} \cdot (-\infty) = \boxed{+\infty} \end{aligned}$$