

6.10 1)

Είναι $\lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty$

Αρα αφού $f(x) \geq \frac{1}{x^2}$

$$\left. \begin{array}{l} f(x) \geq \frac{1}{x^2} \\ \lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = +\infty$$

6.10 2)

Είναι $\lim_{x \rightarrow 2} \frac{3-2x}{(x-2)^2} = \lim_{x \rightarrow 2} (3-2x) \cdot \lim_{x \rightarrow 2} \frac{1}{(x-2)^2} = (3-2 \cdot 2)(+\infty) = -\infty$

Αρα αφού $f(x) \leq \frac{3-2x}{(x-2)^2}$

$$\left. \begin{array}{l} f(x) \leq \frac{3-2x}{(x-2)^2} \\ \lim_{x \rightarrow 2} \frac{3-2x}{(x-2)^2} = -\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = -\infty$$

6.10 3)

Είναι $\lim_{x \rightarrow 3} \frac{2x-6}{(x-3)^3} = \lim_{x \rightarrow 3} \frac{2 \cancel{(x-3)}}{(x-3)^{\cancel{3}^2}} = \lim_{x \rightarrow 3} \frac{2}{(x-3)^2} = 2 \cdot \lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = 2 \cdot (+\infty) = +\infty$

Αρα αφού $f(x) \geq \frac{2x-6}{(x-3)^3}$

$$\left. \begin{array}{l} f(x) \geq \frac{2x-6}{(x-3)^3} \\ \lim_{x \rightarrow 3} \frac{2x-6}{(x-3)^3} = +\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 3} f(x) = +\infty$$

6.10 4)

Είναι

$\lim_{x \rightarrow 0} \frac{x^2 - 2x}{x^4 + x^3} = \lim_{x \rightarrow 0} \frac{\cancel{x}(x-2)}{x^{\cancel{4}^3}(x+1)} = \lim_{x \rightarrow 0} \frac{x-2}{x+1} \cdot \lim_{x \rightarrow 0} \frac{1}{x^2} = \frac{0-2}{0+1} (+\infty) = -2 \cdot (+\infty) = -\infty$

Αρα αφού $f(x) \leq \frac{x^2 - 2x}{x^4 + x^3}$

$$\left. \begin{array}{l} f(x) \leq \frac{x^2 - 2x}{x^4 + x^3} \\ \lim_{x \rightarrow 0} \frac{x^2 - 2x}{x^4 + x^3} = -\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = -\infty$$

6.10 5)

Είναι

$$\lim_{x \rightarrow -1} \frac{x}{x^2 + 2x + 1} = \lim_{x \rightarrow -1} \frac{x}{(x+1)^2} = \lim_{x \rightarrow -1} x \cdot \lim_{x \rightarrow -1} \frac{1}{(x+1)^2} = (-1)(+\infty) = -\infty$$

$$\text{Άρα αφού } f(x) \leq \frac{x}{x^2 + 2x + 1}$$

$$\left. \begin{array}{l} f(x) \leq \frac{x}{x^2 + 2x + 1} \\ \lim_{x \rightarrow -1} \frac{x}{x^2 + 2x + 1} = -\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow -1} f(x) = -\infty$$

6.10 6)

Είναι

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{2x+3}{x^3 - x^2 - x + 1} &\stackrel{\substack{x^3 - x^2 - x + 1: \text{ρίζα στο } x=1 \\ \text{παραγοντοποίηση με Horner}}}{\Rightarrow} \lim_{x \rightarrow 1} \frac{2x+3}{(x-1)^2(x+1)} = \lim_{x \rightarrow 1} \frac{2x+3}{x+1} \cdot \lim_{x \rightarrow 1} \frac{1}{(x-1)^2} \\ &= \frac{2 \cdot 1 + 3}{1+1} \cdot (+\infty) = \frac{5}{2} \cdot (+\infty) = +\infty \end{aligned}$$

$$\text{Άρα αφού } f(x) \geq \frac{2x+3}{x^3 - x^2 - x + 1}$$

$$\left. \begin{array}{l} f(x) \geq \frac{2x+3}{x^3 - x^2 - x + 1} \\ \lim_{x \rightarrow 1} \frac{2x+3}{x^3 - x^2 - x + 1} = +\infty \end{array} \right\} \Rightarrow \lim_{x \rightarrow 1} f(x) = +\infty$$