5.6 1)

α)
$$\lim_{x\to1}\frac{2\left|2x-3\right|-3\left|x+2\right|+7}{\left|-x-2\right|-\left|2x+1\right|}\overset{\substack{\lim\limits_{x\to1}(2x-3)=-1<0\\ \lim\limits_{x\to1}(x+2)=3>0\\ \lim\limits_{x\to1}(-x-2)=-3<0\\ \lim\limits_{x\to1}(2x+1)=3>0}}{=}\lim_{x\to1}\frac{2(-2x+3)-3(x+2)+7}{x+2-(2x+1)}=$$
$$=\lim_{x\to1}\frac{-4x+6-3x-6+7}{x+2-(2x+1)}\lim_{x\to1}\frac{-7x+7}{-x+1}=\lim_{x\to1}\frac{-7\cancel{(x-1)}}{-\cancel{(x-1)}}=7$$

β)
$$\lim_{x\to2}\frac{\left|x^2+x-1\right|-2x-1}{\left|x-3\right|-x+1}\overset{\substack{\lim\limits_{x\to2}(x^2+x-1)=5>0\\ \lim\limits_{x\to2}(x-3)=-1<0}}{=}\lim_{x\to2}\frac{x^2+x-1-2x-1}{-x+3-x+1}=\lim_{x\to2}\frac{x^2-x-2}{-2x+4}=$$
$$=\lim_{x\to2}\frac{\cancel{(x-2)}(x+1)}{-2\cancel{(x-2)}}=\boxed{-\frac{3}{2}}$$

5.6 2)

$$\lim_{x\to-2}\frac{3\left|2x+1\right|+5x+1}{\left|x^2+x-1\right|-1}\overset{\substack{\lim\limits_{x\to-2}(2x+1)=-3<0\\ \lim\limits_{x\to-2}(x^2+x-1)=1<0}}{=}\lim_{x\to-2}\frac{3(-2x-1)+5x+1}{(x^2+x-1)-1}=\lim_{x\to-2}\frac{-6x-3+5x+1}{x^2+x-2}$$

$$x^2+x-2:\Delta=9\Rightarrow x_{1,2}=\frac{-1\pm3}{2}\Rightarrow\begin{cases}x_1=\frac{-1+3}{2}\Rightarrow x_1=1\\ x_2=\frac{-1-3}{2}\Rightarrow x_2=-2\end{cases}$$

$$=\lim_{x\to-2}\frac{-x-2}{x^2+x-2}=\lim_{x\to-2}\frac{-\cancel{(x+2)}}{(x-1)\cancel{(x+2)}}=\frac{-1}{-2-1}=\boxed{\frac{1}{3}}$$

5.6 3)

$$\lim_{x\to3}\frac{\left|x+2\right|-x^2+4}{\left|5-2x\right|-1}\overset{\substack{\lim\limits_{x\to3}(x+2)=5>0\\ \lim\limits_{x\to3}(5-2x)=-1<0}}{=}\lim_{x\to3}\frac{x+2-x^2+4}{-5+2x-1}=\lim_{x\to3}\frac{-x^2+x+6}{2x-6}=$$

$$-x^2+x+6:\Delta=25\Rightarrow x_{1,2}=\frac{-1\pm5}{-2}\Rightarrow\begin{cases}x_1=\frac{-1-5}{-2}\Rightarrow x_1=3\\ x_2=\frac{-1+5}{-2}\Rightarrow x_2=-2\end{cases}$$

$$=\lim_{x\to3}\frac{-\cancel{(x-3)}(x+2)}{2\cancel{(x-3)}}=\frac{-(3+2)}{2}=\boxed{-\frac{5}{2}}$$

5.6 4)

$$\lim_{x\to2}\frac{\left|x^2+x\right|-2x-2}{x^2+x-6}\overset{\lim\limits_{x\to2}(x^2+x)=6>0}{=}\lim_{x\to2}\frac{(x^2+x)-2x-2}{x^2+x-6}=\lim_{x\to2}\frac{x^2-x-2}{x^2+x-6}=$$

$$x^2-x-2:\Delta=9\Rightarrow x_{1,2}=\frac{1\pm3}{2}\Rightarrow\begin{cases}x_1=\frac{1+3}{2}\Rightarrow x_1=2\\ x_2=\frac{1-3}{2}\Rightarrow x_2=-1\end{cases}$$

$$x^2+x-6:\Delta=25\Rightarrow x_{1,2}=\frac{-1\pm5}{2}\Rightarrow\begin{cases}x_1=\frac{-1+5}{2}\Rightarrow x_1=2\\ x_2=\frac{-1-5}{2}\Rightarrow x_2=-3\end{cases}$$

$$=\lim_{x\to2}\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x+3)}=\frac{2+1}{2+3}=\boxed{\frac{3}{5}}$$

5.6 5)

$$\lim_{x\to1}\frac{3-|2x+2|+|x|}{|-x^2-x|-4x+2}\overset{\substack{\lim\limits_{x\to1}(2x+2)=4>0\\ \lim\limits_{x\to1}x=1>0\\ \lim\limits_{x\to1}(-x^2-x)=-2<0}}{=}\lim_{x\to1}\frac{3-(2x+2)+x}{-(-x^2-x)-4x+2}=\lim_{x\to1}\frac{3-2x-2+x}{x^2+x-4x+2}$$

$$=\lim_{x\to1}\frac{-x+1}{x^2-3x+2}\overset{x^2-3x+2:\Delta=1\Rightarrow x_{1,2}=\frac{1\pm3}{2}\Rightarrow\begin{cases}x_1=\frac{3-1}{2}\Rightarrow x_1=1\\ x_1=\frac{3+1}{2}\Rightarrow x_1=2\end{cases}}{=}\lim_{x\to1}\frac{-\cancel{(x-1)}}{\cancel{(x-1)}(x-2)}=\frac{-1}{1-2}=\boxed{1}$$

5.6 6)

$$\lim_{x\to-4}\frac{|x+3|-1}{|x^2+3x+2|+x-2}\overset{\substack{\lim\limits_{x\to-4}(x+3)=-1<0\\ \lim\limits_{x\to-4}(x^2+3x+2)=6>0}}{=}\lim_{x\to-4}\frac{-x-3-1}{x^2+3x+2+x-2}=\lim_{x\to-4}\frac{-x-4}{x^2+4x}=$$

$$=\lim_{x\to-4}\frac{-\cancel{(x+4)}}{x\cancel{(x+4)}}=\frac{-1}{-4}=\boxed{\frac{1}{4}}$$

5.6 7)

$$\lim_{x\to-2}\frac{2x+|x^2+x+2|}{x^2+x-2}\overset{\lim\limits_{x\to-2}(x^2+x+2)=4>0}{=}\lim_{x\to-2}\frac{2x+x^2+x+2}{x^2+x-2}=\lim_{x\to-2}\frac{x^2+3x+2}{x^2+x-2}=$$

$$\overset{\substack{x^2+3x+2:\Delta=1\Rightarrow x_{1,2}=\frac{-3\pm1}{2}\Rightarrow\begin{cases}x_1=\frac{-3-1}{2}\Rightarrow x_1=-2\\ x_2=\frac{-3+1}{2}\Rightarrow x_2=-1\end{cases}\\ x^2+x-2:\Delta=9\Rightarrow x_{1,2}=\frac{1\pm3}{2}\Rightarrow\begin{cases}x_1=\frac{-1-3}{2}\Rightarrow x_1=-2\\ x_2=\frac{-1+3}{2}\Rightarrow x_2=1\end{cases}}}{=}\lim_{x\to-2}\frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-1)}=\frac{-2+1}{-2-1}=\boxed{\frac{1}{3}}$$

5.6 8)

$$\lim_{x\to2}\frac{3|x^2+x-5|-x-1}{x^3+x^2-4x-4}\overset{\lim\limits_{x\to2}(x^2+x-5)=1>0}{=}\lim_{x\to2}\frac{3(x^2+x-5)-x-1}{x^3+x^2-4x-4}=$$

$$=\lim_{x\to2}\frac{3x^2+3x-15-x-1}{x^3+x^2-4x-4}=\lim_{x\to2}\frac{3x^2+2x-16}{x^3+x^2-4x-4}\overset{\substack{3x^2+2x-16:\Delta=196\Rightarrow x_{1,2}=\frac{-2\pm14}{4}\Rightarrow\begin{cases}x_1=\frac{-2-14}{6}\Rightarrow x_1=\frac{-16}{6}=-\frac{8}{3}\\ x_2=\frac{-2+14}{6}\Rightarrow x_2=2\end{cases}\\ x^3+x^2-4x-4:\begin{cases}\text{ρίζα το } x=2\\ \text{παραγοντοποίηση με Horner}\end{cases}}}{=}$$

$$=\lim_{x\to2}\frac{3\left(x+\frac{8}{3}\right)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+3x+2)}=\lim_{x\to2}\frac{3x+8}{x^2+3x+2}=\frac{3\cdot2+8}{2^2+3\cdot2+2}=\frac{14}{12}=\boxed{\frac{7}{6}}$$

5.6 9)

$$\lim_{x\to4}\frac{|2x-5|-x+1}{x-|4-2x|}\overset{\substack{\lim\limits_{x\to4}(2x-5)=3>0\\ \lim\limits_{x\to4}(4-2x)=-4<0}}{=}\lim_{x\to4}\frac{2x-5-x+1}{x-(-4+2x)}=\lim_{x\to4}\frac{x-4}{x+4-2x}$$

$$= \lim_{x \to 4} \frac{x-4}{-x+4} = \lim_{x \to 4} \frac{\cancel{x-4}}{-\cancel{(x-4)}} = \boxed{-1}$$