5.5 1)

α) $\lim\limits_{x\to -3}\dfrac{\sqrt{2x+7}-1}{x^2-9} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{2x+7}+1\\ x^2-9=(x-3)(x+3)}}{=} \lim\limits_{x\to -3}\dfrac{\left(\sqrt{2x+7}-1\right)\left(\sqrt{2x+7}+1\right)}{(x-3)(x+3)\left(\sqrt{2x+7}+1\right)} =$

$$= \lim_{x\to -3}\frac{\left(\sqrt{2x+7}\right)^2-1^2}{(x-3)(x+3)\left(\sqrt{2x+7}+1\right)} = \lim_{x\to -3}\frac{2x+6}{(x-3)(x+3)\left(\sqrt{2x+7}+1\right)} =$$

$$= \lim_{x\to -3}\frac{2\cancel{(x+3)}}{(x-3)\cancel{(x+3)}\left(\sqrt{2x+7}+1\right)} = \frac{2}{(-3-3)\left(\sqrt{2(-3)+7}+1\right)} = \frac{2}{-6\cdot 2} = -\frac{1}{6}$$

β) $\lim\limits_{x\to 9}\dfrac{3-\sqrt{x}}{4-\sqrt{x+7}} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με}\\ (3+\sqrt{x})(4+\sqrt{x+7})}}{=} \lim\limits_{x\to 9}\dfrac{(3-\sqrt{x})(3+\sqrt{x})(4+\sqrt{x+7})}{(4-\sqrt{x+7})(3+\sqrt{x})(4+\sqrt{x+7})} =$

$$= \lim_{x\to 9}\frac{\left(3^2-\sqrt{x}^2\right)\left(4+\sqrt{x+7}\right)}{\left(4^2-\sqrt{x+7}^2\right)\left(3+\sqrt{x}\right)} = \lim_{x\to 9}\frac{(9-x)\left(4+\sqrt{x+7}\right)}{(16-x-7)\left(3+\sqrt{x}\right)} =$$

$$= \lim_{x\to 9}\frac{\cancel{(9-x)}\left(4+\sqrt{x+7}\right)}{\cancel{(9-x)}\left(3+\sqrt{x}\right)} = \lim_{x\to 9}\frac{4+\sqrt{x+7}}{3+\sqrt{x}} = \frac{4+\sqrt{9+7}}{3+\sqrt{9}} = \frac{4+4}{3+3} = \frac{8}{6} = \frac{4}{3}$$

5.5 2)

$$\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{x-1}+2}}{=} \lim_{x\to 5}\frac{\left(\sqrt{x-1}-2\right)\left(\sqrt{x-1}+2\right)}{(x-5)\left(\sqrt{x-1}+2\right)} =$$

$$= \lim_{x\to 5}\frac{\sqrt{x-1}^2-2^2}{(x-5)\left(\sqrt{x-1}+2\right)} = \lim_{x\to 5}\frac{\cancel{x-5}}{\cancel{(x-5)}\left(\sqrt{x-1}+2\right)} = \frac{1}{\sqrt{5-1}+2} = \boxed{\frac{1}{4}}$$

5.5 3)

$$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x^2-4} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{x+2}+2}}{=} \lim_{x\to 2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{(x-2)(x+2)\left(\sqrt{x+2}+2\right)}$$

$$= \lim_{x\to 2}\frac{\sqrt{x+2}^2-2^2}{(x-2)(x+2)\left(\sqrt{x+2}+2\right)} = \lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)\left(\sqrt{x+2}+2\right)} =$$

$$= \frac{1}{(2+2)\left(\sqrt{2+2}+2\right)} = \boxed{\frac{1}{16}}$$

5.5 4)

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2+x-2} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{x+3}+2}}{=} \lim_{x\to 1}\frac{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}{\left(x^2+x-2\right)\left(\sqrt{x+3}+2\right)} =$$

$x^2+x-2:\Delta=9\Rightarrow x_{1,2}=\frac{-1\pm3}{2}\Rightarrow\begin{cases}x_1=\frac{-1+3}{2}\Rightarrow x_1=1\\ x_2=\frac{-1-3}{2}\Rightarrow x_2=-2\end{cases}$

$$=\lim_{x\to1}\frac{\sqrt{x+3}^2-2^2}{(x-1)(x+2)\left(\sqrt{x+3}+2\right)}=$$

$$=\frac{1}{(1+2)\left(\sqrt{1+3}+2\right)}=\boxed{\frac{1}{12}}$$

5.5 5)

$$\lim_{x\to2}\frac{\sqrt{2x+5}-\sqrt{3x+3}}{x^2-4}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{2x+5}+\sqrt{3x+3}}}{=}$$

$$=\lim_{x\to2}\frac{\left(\sqrt{2x+5}-\sqrt{3x+3}\right)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}{\left(x^2-4\right)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}=$$

$$=\lim_{x\to2}\frac{\sqrt{2x+5}^2-\sqrt{3x+3}^2}{(x-2)(x+2)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}=\lim_{x\to2}\frac{2x+5-3x-3}{(x-2)(x+2)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}=$$

$$=\lim_{x\to2}\frac{-x+2}{(x-2)(x+2)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}=\lim_{x\to2}\frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x+2)\left(\sqrt{2x+5}+\sqrt{3x+3}\right)}=$$

$$=\frac{-1}{(2+2)\left(\sqrt{2\cdot2+5}+\sqrt{3\cdot2+3}\right)}=\frac{-1}{4(3+3)}=\boxed{-\frac{1}{24}}$$

5.5 6)

$$\lim_{x\to5}\frac{\sqrt{2x-1}-\sqrt{3x-6}}{x^2-25}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{2x-1}+\sqrt{3x-6}}}{=}$$

$$=\lim_{x\to5}\frac{\left(\sqrt{2x-1}-\sqrt{3x-6}\right)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}{\left(x^2-25\right)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}=\lim_{x\to5}\frac{\sqrt{2x-1}^2-\sqrt{3x-6}^2}{(x-5)(x+5)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}=$$

$$=\lim_{x\to5}\frac{2x-1-3x+6}{(x-5)(x+5)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}=\lim_{x\to5}\frac{-x+5}{(x-5)(x+5)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}=$$

$$=\lim_{x\to5}\frac{-\cancel{(x-5)}}{\cancel{(x-5)}(x+5)\left(\sqrt{2x-1}+\sqrt{3x-6}\right)}=$$

$$=\frac{-1}{(5+5)\left(\sqrt{2\cdot5-1}+\sqrt{3\cdot5-6}\right)}=-\frac{1}{10(3+3)}=\boxed{-\frac{1}{60}}$$

5.5 7)

$$\lim_{x\to3}\frac{\sqrt{x^2+x+4}-4}{x^2-2x-3}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{x^2+x+4}+4}}{=}\lim_{x\to3}\frac{\left(\sqrt{x^2+x+4}-4\right)\left(\sqrt{x^2+x+4}+4\right)}{\left(x^2-2x-3\right)\left(\sqrt{x^2+x+4}+4\right)}=$$

$x^2-2x-3:\Delta=16\Rightarrow x_{1,2}=\frac{2\pm4}{2}\Rightarrow\begin{cases}x_1=\frac{2+4}{2}\Rightarrow x_1=3\\ x_2=\frac{2-4}{2}\Rightarrow x_2=-1\end{cases}$

$$=\lim_{x\to3}\frac{\sqrt{x^2+x+4}^2-4^2}{(x-3)(x+1)\left(\sqrt{x^2+x+4}+4\right)}=$$

$$=\lim_{x\to 3}\frac{x^2+x+4-16}{(x-3)(x+1)\left(\sqrt{x^2+x+4}+4\right)}=\lim_{x\to 3}\frac{x^2+x-12}{(x-3)(x+1)\left(\sqrt{x^2+x+4}+4\right)}=$$

$x^2+x-12:\Delta=49\Rightarrow x_{1,2}=\frac{-1\pm 7}{2}\Rightarrow \begin{cases} x_1=\frac{-1+7}{2}\Rightarrow x_1=3 \\ x_2=\frac{-1-7}{2}\Rightarrow x_2=-4 \end{cases}$

$$=\lim_{x\to 3}\frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}(x+1)\left(\sqrt{x^2+x+4}+4\right)}=$$

$$=\frac{3+4}{(3+1)\left(\sqrt{3^2+3+4}+4\right)}=\frac{7}{4(4+4)}=\boxed{\frac{7}{32}}$$

5.5 8)

$$\lim_{x\to -2}\frac{x^2+x-2}{\sqrt{x^2+5}-3}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με } \sqrt{x^2+5}+3}}{=}\lim_{x\to -2}\frac{\left(x^2+x-2\right)\left(\sqrt{x^2+5}+3\right)}{\left(\sqrt{x^2+5}-3\right)\left(\sqrt{x^2+5}+3\right)}=$$

$x^2+x-2:\Delta=9\Rightarrow x_{1,2}=\frac{-1\pm 3}{2}\Rightarrow \begin{cases} x_1=\frac{-1+3}{2}\Rightarrow x_1=1 \\ x_2=\frac{-1-3}{2}\Rightarrow x_2=-2 \end{cases}$

$$=\lim_{x\to -2}\frac{(x-1)(x+2)\left(\sqrt{x^2+5}+3\right)}{\sqrt{x^2+5}^2-3^2}=$$

$$=\lim_{x\to -2}\frac{(x-1)(x+2)\left(\sqrt{x^2+5}+3\right)}{x^2+5-9}=\lim_{x\to -2}\frac{(x-1)(x+2)\left(\sqrt{x^2+5}+3\right)}{x^2-4}$$

$$=\lim_{x\to -2}\frac{(x-1)\cancel{(x+2)}\left(\sqrt{x^2+5}+3\right)}{(x-2)\cancel{(x+2)}}=\frac{(-2-1)\left(\sqrt{(-2)^2+5}+3\right)}{-2-2}=$$

$$=\frac{(-3)(3+3)}{-4}=\frac{-18}{-4}=\boxed{\frac{9}{2}}$$

5.5 9)

$$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{\sqrt{3x-2}-x}\overset{\substack{\text{πολλαπλασιάζουμε αριθμητή}\\ \text{και παρονομαστή με}\\ \left(\sqrt{x+2}+2\right)\left(\sqrt{3x-2}+x\right)}}{=}\lim_{x\to 2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)\left(\sqrt{3x-2}+x\right)}{\left(\sqrt{3x-2}-x\right)\left(\sqrt{x+2}+2\right)\left(\sqrt{3x-2}+x\right)}=$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x+2}^2-2^2\right)\left(\sqrt{3x-2}+x\right)}{\left(\sqrt{3x-2}^2-x^2\right)\left(\sqrt{x+2}+2\right)}=\lim_{x\to 2}\frac{(x+2-4)\left(\sqrt{3x-2}+x\right)}{\left(3x-2-x^2\right)\left(\sqrt{x+2}+2\right)}=$$

$-x^2+3x-2:\Delta=1\Rightarrow x_{1,2}=\frac{-3\pm 1}{-2}\Rightarrow \begin{cases} x_1=\frac{-3-1}{-2}\Rightarrow x_1=2 \\ x_2=\frac{-3+1}{-2}\Rightarrow x_2=1 \end{cases}$

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}\left(\sqrt{3x-2}+x\right)}{-(x-1)\cancel{(x-2)}\left(\sqrt{x+2}+2\right)}=$$

$$=\frac{\sqrt{3\cdot 2-2}+2}{-(2-1)\left(\sqrt{2+2}+2\right)}=\frac{4}{-4}=\boxed{-1}$$

5.5 10)

$$\lim_{x\to 2}\frac{\sqrt{3x+3}-3}{1-\sqrt{x-1}} \overset{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } \left(\sqrt{3x+3}+3\right)\left(1+\sqrt{x-1}\right)}{=} \lim_{x\to 2}\frac{\left(\sqrt{3x+3}-3\right)\left(\sqrt{3x+3}+3\right)\left(1+\sqrt{x-1}\right)}{\left(1-\sqrt{x-1}\right)\left(\sqrt{3x+3}+3\right)\left(1+\sqrt{x-1}\right)}=$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{3x+3}^2-3^2\right)\left(1+\sqrt{x-1}\right)}{\left(1^2-\sqrt{x-1}^2\right)\left(\sqrt{3x+3}+3\right)}=\lim_{x\to 2}\frac{(3x+3-9)\left(1+\sqrt{x-1}\right)}{(1-x+1)\left(\sqrt{3x+3}+3\right)}=$$

$$=\lim_{x\to 2}\frac{(3x-6)\left(1+\sqrt{x-1}\right)}{(2-x)\left(\sqrt{3x+3}+3\right)}=\lim_{x\to 2}\frac{3\cancel{(x-2)}\left(1+\sqrt{x-1}\right)}{-\cancel{(x-2)}\left(\sqrt{3x+3}+3\right)}=$$

$$=\frac{3\left(1+\sqrt{2-1}\right)}{-\left(\sqrt{3\cdot 2+3}+3\right)}=\frac{6}{-6}=\boxed{-1}$$

5.5 11)

$$\lim_{x\to -1}\frac{\sqrt{2x+3}+x}{\sqrt{x+2}-1} \overset{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } \left(\sqrt{2x+3}-x\right)\left(\sqrt{x+2}+1\right)}{=} \lim_{x\to -1}\frac{\left(\sqrt{2x+3}+x\right)\left(\sqrt{2x+3}-x\right)\left(\sqrt{x+2}+1\right)}{\left(\sqrt{x+2}-1\right)\left(\sqrt{2x+3}-x\right)\left(\sqrt{x+2}+1\right)}=$$

$$=\lim_{x\to -1}\frac{\left(\sqrt{2x+3}^2-x^2\right)\left(\sqrt{x+2}+1\right)}{\left(\sqrt{x+2}^2-1^2\right)\left(\sqrt{2x+3}-x\right)}=\lim_{x\to -1}\frac{\left(2x+3-x^2\right)\left(\sqrt{x+2}+1\right)}{(x+2-1)\left(\sqrt{2x+3}-x\right)}=$$

$$=\lim_{x\to -1}\frac{\left(-x^2+2x+3\right)\left(\sqrt{x+2}+1\right)}{(x+1)\left(\sqrt{2x+3}-x\right)} \overset{-x^2+2x+3:\Delta=16\Rightarrow x_{1,2}=\frac{-2\pm 4}{-2}\Rightarrow \begin{cases} x_1=\frac{-2-4}{-2}\Rightarrow x_1=3 \\ x_2=\frac{-2+4}{-2}\Rightarrow x_2=-1 \end{cases}}{=}$$

$$=\lim_{x\to -1}\frac{-\cancel{(x+1)}(x-3)\left(\sqrt{x+2}+1\right)}{\cancel{(x+1)}\left(\sqrt{2x+3}-x\right)}=\frac{-(-1-3)\left(\sqrt{-1+2}+1\right)}{\sqrt{2\cdot(-1)+3}-(-1)}=\frac{8}{2}=\boxed{4}$$

5.5 12)

$$\lim_{x\to 2}\frac{\sqrt{x^2-x+2}-2}{4-\sqrt{5x+6}} \overset{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } \left(\sqrt{x^2-x+2}+2\right)\left(4+\sqrt{5x+6}\right)}{=}$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x^2-x+2}-2\right)\left(\sqrt{x^2-x+2}+2\right)\left(4+\sqrt{5x+6}\right)}{\left(4-\sqrt{5x+6}\right)\left(\sqrt{x^2-x+2}+2\right)\left(4+\sqrt{5x+6}\right)}=$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x^2-x+2}^2-2^2\right)\left(4+\sqrt{5x+6}\right)}{\left(4^2-\sqrt{5x+6}^2\right)\left(\sqrt{x^2-x+2}+2\right)}=\lim_{x\to 2}\frac{\left(x^2-x+2-4\right)\left(4+\sqrt{5x+6}\right)}{(16-5x-6)\left(\sqrt{x^2-x+2}+2\right)}=$$

$$=\lim_{x\to 2}\frac{\left(x^2-x-2\right)\left(4+\sqrt{5x+6}\right)}{(10-5x)\left(\sqrt{x^2-x+2}+2\right)} \overset{x^2-x-2:\Delta=9\Rightarrow x_{1,2}=\frac{1\pm 3}{2}\Rightarrow \begin{cases} x_1=\frac{1+3}{2}\Rightarrow x_1=2 \\ x_2=\frac{1-3}{2}\Rightarrow x_2=-1 \end{cases}}{=}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}\,(x+1)\left(4+\sqrt{5x+6}\right)}{-5\cancel{(x-2)}\left(\sqrt{x^2-x+2}+2\right)} = \frac{(2+1)\left(4+\sqrt{5\cdot 2+6}\right)}{-5\left(\sqrt{2^2-2+2}+2\right)} = \frac{3(4+4)}{-5(2+2)} = \frac{24}{-20} = \boxed{-\frac{6}{5}}$$