

5.44

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{f(x^2 + x)}{x^2 - x} & \stackrel{\substack{\text{δαιρούμε} \\ \text{με } x^2 + x}}{=} \lim_{x \rightarrow 0} \frac{\frac{f(x^2 + x)}{x^2 + x}}{\frac{x^2 - x}{x^2 + x}} = \lim_{x \rightarrow 0} \frac{f(x^2 + x)}{x^2 + x} \lim_{x \rightarrow 0} \frac{f(x^2 + x)}{x^2 + x} \stackrel{\substack{\text{θέτουμε } y = x^2 + x \\ \text{όταν } x \rightarrow 0, y \rightarrow 0}}{=} \lim_{y \rightarrow 0} \frac{f(y)}{y} \stackrel{\lim_{x \rightarrow 0} \frac{f(x)}{x} = 5}{=} 5 \\
 & = \frac{5}{0 - 1} = \boxed{-5} \\
 & \quad 0 + 1
 \end{aligned}$$