5.42

Θέτουμε $h(x)=\dfrac{g(x)\eta\mu x-xf(x)}{2x}$ (1) και $\varphi(x)=\dfrac{g(x)\eta\mu x+xf(x)}{\eta\mu x}$ (2)

Προφανώς είναι $\lim\limits_{x\to\alpha}h(x)=1$ και $\lim\limits_{x\to\alpha}\varphi(x)=\dfrac{1}{2}$

Λύνουμε (σαν σύστημα) τις (1) και (2)

$$\begin{cases}\dfrac{g(x)\eta\mu x-xf(x)}{2x}=h(x)\\ \dfrac{g(x)\eta\mu x+xf(x)}{\eta\mu x}=\varphi(x)\end{cases}\Rightarrow\begin{cases}g(x)\eta\mu x-xf(x)=2xh(x)\\ g(x)\eta\mu x+xf(x)=\eta\mu x\varphi(x)\end{cases}\quad(3)$$

$(3)\overset{\text{προσθέτουμε κατά μέλη}}{\Rightarrow}2g(x)\eta\mu x=2xh(x)+\eta\mu x\varphi(x)\Rightarrow\boxed{g(x)=\dfrac{2xh(x)+\eta\mu x\cdot\varphi(x)}{2\eta\mu x}}$ (4)

$(3)\overset{\text{αφαιρούμε κατά μέλη}}{\Rightarrow}-2xf(x)=2xh(x)-\eta\mu x\varphi(x)\Rightarrow\boxed{f(x)=\dfrac{2xh(x)-\eta\mu x\cdot\varphi(x)}{-2x}}$ (5)

Οπότε

$$\boxed{\lim_{x\to0}f(x)}\overset{(5)}{=}\lim_{x\to0}\frac{2xh(x)-\eta\mu x\cdot\varphi(x)}{-2x}=$$

$$=\lim_{x\to0}\frac{2\cancel{x}h(x)}{-2\cancel{x}}+\frac{\eta\mu x\cdot\varphi(x)}{2x}\overset{\lim\limits_{x\to0}h(x)=1,\ \lim\limits_{x\to0}\varphi(x)=\frac{1}{2}}{=}\overset{\lim\limits_{x\to0}\frac{\eta\mu x}{x}=1}{=}-1+\frac{1}{4}=\boxed{-\frac{3}{4}}$$

και

$$\boxed{\lim_{x\to0}g(x)}\overset{(4)}{=}\lim_{x\to0}\frac{2xh(x)+\eta\mu x\cdot\varphi(x)}{2\eta\mu x}=$$

$$=\lim_{x\to0}\frac{\cancel{2}xh(x)}{\cancel{2}\eta\mu x}+\frac{\cancel{\eta\mu x}\cdot\varphi(x)}{\cancel{2}\cancel{\eta\mu x}}\overset{\lim\limits_{x\to0}h(x)=1,\ \lim\limits_{x\to0}\varphi(x)=\frac{1}{2}}{=}\overset{\lim\limits_{x\to0}\frac{x}{\eta\mu x}=1}{=}1\cdot1+\frac{1}{4}=\boxed{\frac{5}{4}}$$