

5.4 1)

$$\alpha) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - 4x + 3} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)(x-3)} = \frac{1+1}{1-3} = \boxed{-1}$$

$x^2 - 4x + 3 : \Delta = 4 \Rightarrow x_{1,2} = \frac{4 \pm 2}{2} \Rightarrow \begin{cases} x_1 = \frac{4+2}{2} \Rightarrow x_1 = 3 \\ x_1 = \frac{4-2}{2} \Rightarrow x_1 = 1 \end{cases}$

$$\lim_{x \rightarrow -1} \frac{x^4 + 3x^3 - 2x^2 + x + 5}{x^3 + 2x^2 - 1} \stackrel{\text{ρίζα το } x = -1 \text{ παραγοντοποίηση με Horner}}{=} \lim_{x \rightarrow -1} \frac{(x+1)(x^3 + 2x^2 - 4x + 5)}{(x+1)(x^2 + x - 1)} =$$

$$= \frac{(-1)^3 + 2 \cdot (-1)^2 - 4(-1) + 5}{(-1)^2 + (-1) - 1} = \frac{-1 + 2 + 4 + 5}{1 - 1 - 1} = \boxed{-10}$$

5.4 2)

$$\lim_{x \rightarrow -4} \frac{x^2 + 3x - 4}{x + 4} = \lim_{x \rightarrow -4} \frac{(x-1)(x+4)}{x+4} = -4 - 1 = \boxed{-5}$$

$\Delta = 25 \Rightarrow x_{1,2} = \frac{-3 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{-3+5}{2} \Rightarrow x_1 = 1 \\ x_1 = \frac{-3-5}{2} \Rightarrow x_1 = -4 \end{cases}$

5.4 3)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{2x^2 - 18} \stackrel{2x^2 - 18 = 2(x^2 - 9) = 2(x-3)(x+3)}{=} \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{2(x-3)(x+3)} = \frac{3+2}{2(3+3)} = \boxed{\frac{5}{12}}$$

$x^2 - x - 6 : \Delta = 25 \Rightarrow x_{1,2} = \frac{1 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{1+5}{2} \Rightarrow x_1 = 3 \\ x_2 = \frac{1-5}{2} \Rightarrow x_2 = -2 \end{cases}$

5.4 4)

$$\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 6x + 5} = \lim_{x \rightarrow 1} \frac{(x-1)(x+3)}{(x-1)(x-5)} = \frac{1+3}{1-5} = \boxed{-1}$$

$x^2 + 2x - 3 : \Delta = 16 \Rightarrow x_{1,2} = \frac{-2 \pm 4}{2} \Rightarrow \begin{cases} x_1 = \frac{-2+4}{2} \Rightarrow x_1 = 1 \\ x_1 = \frac{-2-4}{2} \Rightarrow x_1 = -3 \end{cases}$
 $x^2 - 6x + 5 : \Delta = 16 \Rightarrow x_{1,2} = \frac{6 \pm 4}{2} \Rightarrow \begin{cases} x_1 = \frac{6+4}{2} \Rightarrow x_1 = 5 \\ x_1 = \frac{6-4}{2} \Rightarrow x_1 = 1 \end{cases}$

5.4 5)

$$\lim_{x \rightarrow -2} \frac{2x^2 + 3x - 2}{x^2 + x - 2} = \lim_{x \rightarrow -2} \frac{2(x+2)(x-\frac{1}{2})}{(x+2)(x-1)} =$$

$$= \lim_{x \rightarrow -2} \frac{2x-1}{x-1} = \frac{2(-2)-1}{-2-1} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$$

$2x^2 + 3x - 2 : \Delta = 25 \Rightarrow x_{1,2} = \frac{-3 \pm 5}{4} \Rightarrow \begin{cases} x_1 = \frac{-3+5}{4} \Rightarrow x_1 = \frac{2}{4} = \frac{1}{2} \\ x_1 = \frac{-3-5}{4} \Rightarrow x_1 = -2 \end{cases}$
 $x^2 + x - 2 : \Delta = 9 \Rightarrow x_{1,2} = \frac{-1 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{-1+3}{2} \Rightarrow x_1 = 1 \\ x_1 = \frac{-1-3}{2} \Rightarrow x_1 = -2 \end{cases}$

5.4 6)

$$\begin{aligned}
 & x^3 - 1 = (x-1)(x^2 + x + 1) \\
 & 2x^2 + 3x - 5: \Delta = 49 \Rightarrow x_{1,2} = \frac{-3 \pm 7}{4} \Rightarrow \begin{cases} x_1 = \frac{-3-7}{4} \Rightarrow x_1 = -\frac{10}{4} = -\frac{5}{2} \\ x_2 = \frac{-3+7}{4} \Rightarrow x_2 = 1 \end{cases} \\
 & \lim_{x \rightarrow 1} \frac{x^3 - 1}{2x^2 + 3x - 5} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{2\cancel{(x-1)}\left(x + \frac{5}{2}\right)} = \\
 & = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{2x + 5} = \frac{1^2 + 1 + 1}{2 \cdot 1 + 5} = \boxed{\frac{3}{7}}
 \end{aligned}$$

5.4 7)

$$\begin{aligned}
 & \lim_{x \rightarrow 2} \frac{x^3 - x^2 - x - 2}{x^2 - 4} \stackrel{x^3 - x^2 - x - 2: \begin{cases} \text{ρίζα το } x=2 \\ \text{παράγοντοποίηση με Horner} \end{cases}}{=} \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + x + 1)}{\cancel{(x-2)}(x + 2)} = \frac{2^2 + 2 + 1}{2 + 2} = \boxed{\frac{7}{4}}
 \end{aligned}$$

5.4 8)

$$\begin{aligned}
 & \lim_{x \rightarrow 2} \frac{2x^3 - 3x^2 - x - 2}{2x^2 - 8} \stackrel{2x^3 - 3x^2 - x - 2: \begin{cases} \text{ρίζα το } x=2 \\ \text{παράγοντοποίηση με Horner} \end{cases}}{=} \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(2x^2 + x + 1)}{2\cancel{(x-2)}(x + 2)} = \\
 & = \frac{2 \cdot 2^2 + 2 + 1}{2(2 + 2)} = \boxed{\frac{11}{8}}
 \end{aligned}$$

5.4 9)

$$\begin{aligned}
 & \lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{x^2 + x - 2} \stackrel{x^3 - 2x + 1: \begin{cases} \text{ρίζα το } x=1 \\ \text{παράγοντοποίηση με Horner} \end{cases}}{=} \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x^2 + x - 1)}{\cancel{(x-1)}(x + 2)} = \\
 & = \lim_{x \rightarrow 1} \frac{x^2 + x - 1}{x + 2} = \frac{1^2 + 1 - 1}{1 + 2} = \boxed{\frac{1}{3}}
 \end{aligned}$$

5.4 10)

$$\begin{aligned}
 & \lim_{x \rightarrow -2} \frac{2x^2 + x - 6}{x^3 + 2x^2 - 2x - 4} \stackrel{2x^2 + x - 6: \Delta = 49 \Rightarrow x_{1,2} = \frac{-1 \pm 7}{4} \Rightarrow \begin{cases} x_1 = \frac{-1+7}{4} \Rightarrow x_1 = \frac{6}{4} = \frac{3}{2} \\ x_2 = \frac{-1-7}{4} \Rightarrow x_2 = -2 \end{cases}}{=} \lim_{x \rightarrow -2} \frac{2\cancel{(x+2)}\left(x - \frac{3}{2}\right)}{\cancel{(x+2)}(x^2 - 2)} = \\
 & = \lim_{x \rightarrow -2} \frac{2x - 3}{x^2 - 2} = \frac{2(-2) - 3}{(-2)^2 - 2} = \boxed{-\frac{7}{2}}
 \end{aligned}$$

5.4 11)

$$\begin{aligned}
 & \lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6} \stackrel{\text{ρίζα το } x=2 \text{ παράγοντοποίηση με Horner}}{=} \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + 5x + 1)}{\cancel{(x-2)}(x^2 + 2x + 3)} = \frac{2^2 + 5 \cdot 2 + 1}{2^2 + 2 \cdot 2 + 3} = \boxed{\frac{15}{11}}
 \end{aligned}$$

5.4 12)

$$\begin{aligned}
 & \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3} \stackrel{\text{ρίζα το } x=1 \text{ παράγοντοποίηση με Horner}}{=} \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x^2 + x - 2)}{\cancel{(x-1)}(x^3 + x^2 + x - 3)} \stackrel{\text{ρίζα το } x=1 \text{ παράγοντοποίηση με Horner}}{=}
 \end{aligned}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x^2+2x+3)} = \frac{1+2}{1^2+2 \cdot 1+3} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

5.4

13)

$$\lim_{x \rightarrow 1} \frac{2x^4-3x^3+5x^2-2x-2}{x^3+3x^2-2x-2} \overset{\substack{\text{ρίζα το } x=1 \\ \text{παράγοντοποίηση με Horner}}}{=} \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(2x^3-x^2+4x+2)}{\cancel{(x-1)}(x^2+4x+2)} =$$

$$= \lim_{x \rightarrow 1} \frac{2x^3-x^2+4x+2}{x^2+4x+2} = \frac{2 \cdot 1^3-1^2+4 \cdot 1+2}{1^2+4 \cdot 1+2} = \frac{7}{7} = \boxed{1}$$