

5.26 1)

$$\alpha) \lim_{x \rightarrow 0} \frac{\eta\mu 7x}{5x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 7}}{=} \lim_{x \rightarrow 0} \frac{7\eta\mu 7x}{7 \cdot 5x} = \lim_{x \rightarrow 0} \frac{7}{5} \frac{\eta\mu 7x}{7x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 7x}{7x} = 1}{=} \frac{7}{5} \cdot 1 = \boxed{\frac{7}{5}}$$

$$\beta) \lim_{x \rightarrow 0} \frac{\eta\mu 6x}{\eta\mu 2x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 6x \cdot 2x}}{=} \lim_{x \rightarrow 0} \frac{\eta\mu 6x}{6x} \cdot \frac{\cancel{6x}}{\cancel{2x}} \cdot \frac{2x}{\eta\mu 2x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 6x}{6x} = \lim_{x \rightarrow 0} \frac{2x}{\eta\mu 2x} = 1}{=} 1 \cdot \frac{6}{2} \cdot 1 = \boxed{3}$$

$$\gamma) \lim_{x \rightarrow 0} \frac{2\eta\mu x}{x^2 + x} = \lim_{x \rightarrow 0} \frac{2\eta\mu x}{x(x+1)} = \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \cdot \lim_{x \rightarrow 0} \frac{2}{x+1} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu x}{x} = 1}{=} 1 \cdot \frac{2}{0+1} = \frac{2}{1} = \boxed{2}$$

$$\delta) \lim_{x \rightarrow 0} \frac{\eta\mu x}{\sqrt{x+4}-2} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } \sqrt{x+4}+2}}{=} \lim_{x \rightarrow 0} \frac{(\sqrt{x+4}+2)\eta\mu x}{(\sqrt{x+4}-2)(\sqrt{x+4}+2)} =$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x+4}+2)\eta\mu x}{\sqrt{x+4}^2 - 2^2} = \lim_{x \rightarrow 0} \frac{(\sqrt{x+4}+2)\eta\mu x}{x + \cancel{4} - \cancel{4}} = \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \cdot \lim_{x \rightarrow 0} (\sqrt{x+4}+2) \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu x}{x} = 1}{=} 1 \cdot (\sqrt{0+4}+2) = \boxed{4}$$

5.26 2)

$$\lim_{x \rightarrow 0} \frac{\eta\mu 3x}{x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 3}}{=} \lim_{x \rightarrow 0} \frac{3\eta\mu 3x}{3x} = \lim_{x \rightarrow 0} 3 \frac{\eta\mu 3x}{3x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 3x}{3x} = 1}{=} 3 \cdot 1 = \boxed{3}$$

5.26 3)

$$\lim_{x \rightarrow 0} \frac{8x}{\eta\mu 2x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 2}}{=} \lim_{x \rightarrow 0} \frac{2 \cdot 8x}{2\eta\mu 2x} = \lim_{x \rightarrow 0} \frac{8}{2} \frac{2x}{\eta\mu 2x} \stackrel{\lim_{x \rightarrow 0} \frac{2x}{\eta\mu 2x} = 1}{=} 4 \cdot 1 = \boxed{4}$$

5.26 4)

$$\lim_{x \rightarrow 0} \frac{\eta\mu 4x}{\eta\mu 6x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 4x \cdot 6x}}{=} \lim_{x \rightarrow 0} \frac{\eta\mu 4x}{4x} \cdot \frac{\cancel{4x}}{\cancel{6x}} \cdot \frac{6x}{\eta\mu 6x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 4x}{4x} = \lim_{x \rightarrow 0} \frac{6x}{\eta\mu 6x} = 1}{=} 1 \cdot \frac{4}{6} \cdot 1 = \boxed{\frac{2}{3}}$$

5.26 5)

$$\lim_{x \rightarrow 0} \frac{\eta\mu 5x}{\eta\mu 2x} \stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 5x \cdot 2x}}{=} \lim_{x \rightarrow 0} \frac{\eta\mu 5x}{5x} \cdot \frac{\cancel{5x}}{\cancel{2x}} \cdot \frac{2x}{\eta\mu 2x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 5x}{5x} = \lim_{x \rightarrow 0} \frac{2x}{\eta\mu 2x} = 1}{=} 1 \cdot \frac{5}{2} \cdot 1 = \boxed{\frac{5}{2}}$$

5.26 6)

$$\lim_{x \rightarrow 0} \frac{\varepsilon\phi x}{x} = \lim_{x \rightarrow 0} \frac{\eta\mu x}{\sigma\upsilon\nu x} = \lim_{x \rightarrow 0} \frac{\eta\mu x}{x \cdot \sigma\upsilon\nu x} = \lim_{x \rightarrow 0} \frac{1}{\sigma\upsilon\nu x} \cdot \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \stackrel{\lim_{x \rightarrow 0} \frac{1}{\sigma\upsilon\nu 3x} = 1}{=} 1 \cdot 1 = 1$$

5.26 7)

$$\lim_{x \rightarrow 0} \frac{\varepsilon\phi 3x}{x} = \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{\sigma\upsilon\nu 3x} = \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{x \cdot \sigma\upsilon\nu 3x} = \lim_{x \rightarrow 0} \frac{1}{\sigma\upsilon\nu 3x} \cdot \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{x} \stackrel{\lim_{x \rightarrow 0} \frac{1}{\sigma\upsilon\nu 3x} = 1}{=} 1 \cdot \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{x}$$

$$\stackrel{\substack{\text{πολλαπλασιάζουμε αριθμητή} \\ \text{και παρονομαστή με } 3}}{=} \lim_{x \rightarrow 0} \frac{3\eta\mu 3x}{3x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 3x}{3x} = 1}{=} 3 \cdot 1 = \boxed{3}$$

5.26 8)

$$\lim_{x \rightarrow 0} \frac{5x}{\varepsilon\phi 3x} = \lim_{x \rightarrow 0} \frac{5x}{\frac{\eta\mu 3x}{\sigma\upsilon\nu 3x}} = \lim_{x \rightarrow 0} \frac{5x\sigma\upsilon\nu 3x}{\eta\mu 3x} = \lim_{x \rightarrow 0} (\sigma\upsilon\nu 3x) \cdot \lim_{x \rightarrow 0} \frac{5x}{\eta\mu 3x} \stackrel{\lim_{x \rightarrow 0} \sigma\upsilon\nu 3x = 1}{=} \lim_{x \rightarrow 0} \frac{5x}{\eta\mu 3x} =$$

πολλαπλασιάζουμε αριθμητή και παρονομαστή με 3

$$= \lim_{x \rightarrow 0} \frac{5 \cdot 3x}{3\eta\mu 3x} = \frac{5}{3} \lim_{x \rightarrow 0} \frac{3x}{\eta\mu 3x} \stackrel{\lim_{x \rightarrow 0} \frac{3x}{\eta\mu 3x} = 1}{=} \frac{5}{3} \cdot 1 = \boxed{\frac{5}{3}}$$

5.26 9)

$$\lim_{x \rightarrow 0} \frac{3x^2 - x}{\eta\mu x} = \lim_{x \rightarrow 0} \frac{x(3x - 1)}{\eta\mu x} = \lim_{x \rightarrow 0} \frac{x}{\eta\mu x} \cdot \lim_{x \rightarrow 0} (3x - 1) \stackrel{\lim_{x \rightarrow 0} \frac{x}{\eta\mu x} = 1}{=} 1 \cdot (3 \cdot 0 - 1) = \boxed{-1}$$

5.26 10)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + \eta\mu x} - \sqrt{1 - \eta\mu x}}{x} \stackrel{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } \sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x}}{=} \\ = \lim_{x \rightarrow 0} \frac{(\sqrt{1 + \eta\mu x} - \sqrt{1 - \eta\mu x})(\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x})}{x(\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x})} = \lim_{x \rightarrow 0} \frac{\sqrt{1 + \eta\mu x}^2 - \sqrt{1 - \eta\mu x}^2}{x(\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x})} = \\ = \lim_{x \rightarrow 0} \frac{\cancel{1} + \eta\mu x - \cancel{1} + \eta\mu x}{x(\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x})} = \lim_{x \rightarrow 0} \frac{2\eta\mu x}{x(\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x})} = \\ = \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \cdot \lim_{x \rightarrow 0} \frac{2}{\sqrt{1 + \eta\mu x} + \sqrt{1 - \eta\mu x}} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu x}{x} = 1}{=} 1 \cdot \frac{2}{\sqrt{1 + 0} + \sqrt{1 + 0}} = \frac{2}{2} = \boxed{1}$$

5.26 11)

$$\lim_{x \rightarrow 0} \frac{\eta\mu^2 x - 3x}{\sqrt{x+1} - 1} \stackrel{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } \sqrt{x+1} + 1}{=} \lim_{x \rightarrow 0} \frac{(\eta\mu^2 x - 3x)(\sqrt{x+1} + 1)}{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)} = \\ = \lim_{x \rightarrow 0} \frac{(\eta\mu^2 x - 3x)(\sqrt{x+1} + 1)}{\sqrt{x+1}^2 - 1^2} = \lim_{x \rightarrow 0} \frac{(\eta\mu^2 x - 3x)(\sqrt{x+1} + 1)}{x + \cancel{1} - \cancel{1}} = \\ = \lim_{x \rightarrow 0} \frac{(\eta\mu^2 x - 3x)(\sqrt{x+1} + 1)}{x} = \lim_{x \rightarrow 0} \frac{\eta\mu^2 x - 3x}{x} \cdot \lim_{x \rightarrow 0} (\sqrt{x+1} + 1) = \\ = \lim_{x \rightarrow 0} \left(\frac{\eta\mu^2 x}{x} - \frac{3\cancel{x}}{\cancel{x}} \right) \cdot \lim_{x \rightarrow 0} (\sqrt{x+1} + 1) \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu^2 x}{x} = \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \cdot \eta\mu x = 1 \cdot 0 = 0}{=} (0 - 3)(\sqrt{1+1} + 1) = -6$$

5.26 12)

$$\lim_{x \rightarrow 0} \frac{2\eta\mu 4x}{2x - 3\eta\mu 2x} \stackrel{\text{διαφορούμε αριθμητή και παρονομαστή με } x}{=} \lim_{x \rightarrow 0} \frac{\frac{2\eta\mu 4x}{x}}{\frac{2x - 3\eta\mu 2x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{4 \cdot 2\eta\mu 4x}{4x}}{\frac{2\cancel{x} - 2 \cdot 3\eta\mu 2\cancel{x}}{\cancel{x}}} = \lim_{x \rightarrow 0} \frac{4 \cdot 2}{2 - 2 \cdot 3} = \frac{8}{-4} = \boxed{-2}$$

5.26 13)

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{3\eta\mu 2x + 2\eta\mu 3x}{7x - \eta\mu 5x} & \stackrel{\text{διαφορούμε αριθμητή και παρονομαστή με } x}{=} \lim_{x \rightarrow 0} \frac{\frac{3\eta\mu 2x + 2\eta\mu 3x}{x}}{\frac{7x - \eta\mu 5x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{3\eta\mu 2x}{x} + \frac{2\eta\mu 3x}{x}}{\frac{7x}{x} - \frac{\eta\mu 5x}{x}} = \\
 & = \lim_{x \rightarrow 0} \frac{\frac{2 \cdot 3\eta\mu 2x}{2x} + \frac{3 \cdot 2\eta\mu 3x}{3x}}{\frac{7}{1} - \frac{\eta\mu 5x}{5x}} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 2x}{2x} = \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{3x} = \lim_{x \rightarrow 0} \frac{\eta\mu 5x}{5x} = 1}{=} \frac{2 \cdot 3 + 3 \cdot 2}{7 - 5} = \boxed{6}
 \end{aligned}$$

5.26 14)

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\epsilon\phi 3x - x}{\eta\mu 8x + x^2 - 6x} & \stackrel{\text{διαφορούμε αριθμητή και παρονομαστή με } x}{=} \lim_{x \rightarrow 0} \frac{\frac{\epsilon\phi 3x - x}{x}}{\frac{\eta\mu 8x + x^2 - 6x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{\epsilon\phi 3x}{x} - \frac{x}{x}}{\frac{\eta\mu 8x}{x} + \frac{x^2}{x} - \frac{6x}{x}} = \\
 & \stackrel{\lim_{x \rightarrow 0} \frac{\epsilon\phi 3x}{3x} = \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{3x} \cdot \frac{1}{\sin 3x} = 1 \cdot 1 = 1}{=} \lim_{x \rightarrow 0} \frac{3 \frac{\epsilon\phi 3x}{3x} - 1}{8 \frac{\eta\mu 8x}{8x} + x - 6} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu 8x}{8x} = 1}{=} \frac{3 \cdot 1 - 1}{8 \cdot 1 + 0 - 6} = \frac{2}{2} = \boxed{1}
 \end{aligned}$$

5.26 15)

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{1 - \sigma\upsilon\nu x}{x^2} & \stackrel{\text{πολλαπλασιάζουμε αριθμητή και παρονομαστή με } 1 + \sigma\upsilon\nu x}{=} \lim_{x \rightarrow 0} \frac{(1 - \sigma\upsilon\nu x)(1 + \sigma\upsilon\nu x)}{x^2(1 + \sigma\upsilon\nu x)} = \lim_{x \rightarrow 0} \frac{1 - \sigma\upsilon\nu^2 x}{x^2(1 + \sigma\upsilon\nu x)} = \\
 & = \lim_{x \rightarrow 0} \frac{\eta\mu^2 x}{x^2(1 + \sigma\upsilon\nu x)} = \lim_{x \rightarrow 0} \left(\frac{\eta\mu x}{x} \right)^2 \cdot \lim_{x \rightarrow 0} \frac{1}{1 + \sigma\upsilon\nu x} \stackrel{\lim_{x \rightarrow 0} \frac{\eta\mu x}{x} = 1}{=} 1 \cdot \frac{1}{1 + 1} = \boxed{\frac{1}{2}}
 \end{aligned}$$