

5.23 1)

$$\lim_{x \rightarrow 9} \frac{x - \sqrt{x} - 6}{\sqrt{x} - 3} \stackrel{\substack{\text{θέτουμε } y = \sqrt{x} \\ \text{όταν } x \rightarrow 9, y \rightarrow 3}}{=} \lim_{y \rightarrow 3} \frac{y^2 - y - 6}{y - 3} \stackrel{\Delta = 25, y_1 = -2, y_2 = 3}{=} \\ = \lim_{y \rightarrow 3} \frac{(y+2)(\cancel{y-3})}{\cancel{y-3}} = 3 + 2 = \boxed{5}$$

5.23 2)

$$\lim_{x \rightarrow 16} \frac{3\sqrt{x} - x + 4}{\sqrt{x} - 4} \stackrel{\substack{\text{θέτουμε } y = \sqrt{x} \\ \text{όταν } x \rightarrow 16, y \rightarrow 4}}{=} \lim_{y \rightarrow 4} \frac{3y - y^2 + 4}{y - 4} \stackrel{-y^2 + 3y + 4: \Delta = 25 \Rightarrow y_{1,2} = \frac{-3 \pm 5}{-2} \Rightarrow \begin{cases} y_1 = \frac{-3+5}{-2} \Rightarrow y_1 = -1 \\ y_2 = \frac{-3-5}{-2} \Rightarrow y_2 = 4 \end{cases}}{=} \\ = \lim_{y \rightarrow 4} \frac{-(y+1)(\cancel{y-4})}{\cancel{y-4}} = -(4+1) = \boxed{-5}$$

5.23 3)

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} + x - 6}{2 - x + \sqrt{x}} \stackrel{\substack{\text{θέτουμε } y = \sqrt{x} \\ \text{όταν } x \rightarrow 4, y \rightarrow 2}}{=} \lim_{y \rightarrow 2} \frac{y + y^2 - 6}{2 - y^2 + y} \stackrel{\substack{y^2 + y - 6: \Delta = 25 \Rightarrow y_{1,2} = \frac{-1 \pm 5}{2} \Rightarrow \begin{cases} y_1 = \frac{-1+5}{2} \Rightarrow y_1 = 2 \\ y_2 = \frac{-1-5}{2} \Rightarrow y_2 = -3 \end{cases} \\ -y^2 + y + 2: \Delta = 9 \Rightarrow x_{1,2} = \frac{-1 \pm 3}{-2} \Rightarrow \begin{cases} y_1 = \frac{-1+3}{-2} \Rightarrow y_1 = -1 \\ y_2 = \frac{-1-3}{-2} \Rightarrow y_2 = 2 \end{cases}}{=} \\ = \lim_{y \rightarrow 2} \frac{(\cancel{y-2})(y+3)}{-(y+1)(\cancel{y-2})} = \frac{2+3}{-(2+1)} = \boxed{-\frac{5}{3}}$$

5.23 4)

$$\lim_{x \rightarrow 1} \frac{x\sqrt{x} - 1}{x^2 - \sqrt{x}} \stackrel{\substack{\text{θέτουμε } y = \sqrt{x} \\ \text{όταν } x \rightarrow 1, y \rightarrow 1}}{=} \lim_{y \rightarrow 1} \frac{y^2 y - 1}{(y^2)^2 - y} = \lim_{y \rightarrow 1} \frac{y^3 - 1}{y^4 - y} = \lim_{y \rightarrow 1} \frac{\cancel{y^3 - 1}}{y(\cancel{y^3 - 1})} = \frac{1}{1} = \boxed{1}$$

5.23 5)

$$\lim_{x \rightarrow 8} \frac{x - 8}{\sqrt[3]{x} - 2} \stackrel{\substack{\text{θέτουμε } y = \sqrt[3]{x} \\ \text{όταν } x \rightarrow 8, y \rightarrow 2}}{=} \lim_{y \rightarrow 2} \frac{y^3 - 8}{y - 2} = \lim_{y \rightarrow 2} \frac{(\cancel{y-2})(y^2 + 2y + 4)}{\cancel{y-2}} = 2^2 + 2 \cdot 2 + 4 = \boxed{12}$$