

5.20 1)

Έχουμε

$$\left| \frac{f(x) - 3}{x - 1} \right| \leq 7 \Rightarrow \frac{|f(x) - 3|}{|x - 1|} \leq 7 \Rightarrow |f(x) - 3| \leq 7|x - 1| \Rightarrow$$

$$\Rightarrow -7|x - 1| \leq f(x) - 3 \leq 7|x - 1| \Rightarrow -7|x - 1| + 3 \leq f(x) \leq 7|x - 1| + 3$$

Οπότε

$$\left. \begin{array}{l} \lim_{x \rightarrow 1} (-7|x - 1| + 3) = -7 \cdot 0 + 3 = 3 \\ \lim_{x \rightarrow 1} (7|x - 1| + 3) = 7 \cdot 0 + 3 = 3 \\ -7|x - 1| + 3 \leq f(x) \leq 7|x - 1| + 3 \end{array} \right\} \begin{array}{l} \text{κριτήριο παρεμβολής} \\ \Rightarrow \lim_{x \rightarrow 1} f(x) = 3 \end{array}$$

5.20 2)

Έχουμε

$$\left| \frac{f(x) + 5}{x} \right| \leq 3 \Rightarrow \frac{|f(x) + 5|}{|x|} \leq 3 \Rightarrow |f(x) + 5| \leq 3|x| \Rightarrow$$

$$\Rightarrow -3|x| \leq f(x) + 5 \leq 3|x| \Rightarrow -3|x| - 5 \leq f(x) \leq 3|x| - 5$$

Οπότε

$$\left. \begin{array}{l} \lim_{x \rightarrow 0} (-3|x| - 5) = -3 \cdot 0 - 5 = -5 \\ \lim_{x \rightarrow 0} (3|x| - 5) = 3 \cdot 0 - 5 = -5 \\ -3|x| - 5 \leq f(x) \leq 3|x| - 5 \end{array} \right\} \begin{array}{l} \text{κριτήριο παρεμβολής} \\ \Rightarrow \lim_{x \rightarrow 0} f(x) = -5 \end{array}$$