

5.13 1)

Πρέπει

$$\boxed{\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)} \Rightarrow \lim_{x \rightarrow 2^-} (3\alpha x - \beta) = \lim_{x \rightarrow 2^+} (4\alpha x - 1) \Rightarrow$$

$$\Rightarrow 6\alpha - \beta = 8\alpha - 1 \Rightarrow 2\alpha + \beta = 1 \quad (1)$$

και

$$\boxed{\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x)} \Rightarrow \lim_{x \rightarrow 5^-} (4\alpha x - 1) = \lim_{x \rightarrow 5^+} (x - 3\beta - 17) \Rightarrow$$

$$\Rightarrow 20\alpha - 1 = 5 - 3\beta - 17 \Rightarrow 20\alpha + 3\beta = -11 \quad (2)$$

Λύνοντας το σύστημα των (1) και (2) βρίσκουμε $\boxed{\alpha = -1 \text{ και } \beta = 3}$

5.13 2)

Πρέπει

$$\boxed{\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x)} \Rightarrow \lim_{x \rightarrow -1^-} (2\alpha x^2 + \beta x + 1) = \lim_{x \rightarrow -1^+} (3x - 2) \Rightarrow$$

$$\Rightarrow 2\alpha - \beta + 1 = -3 - 2 \Rightarrow 2\alpha - \beta = -6 \quad (1)$$

και

$$\boxed{\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)} \Rightarrow \lim_{x \rightarrow 1^-} (3x - 2) = \lim_{x \rightarrow 1^+} (\alpha x - \beta) \Rightarrow$$

$$\Rightarrow 3 - 2 = \alpha - \beta \Rightarrow \alpha - \beta = 1 \quad (2)$$

Λύνοντας το σύστημα των (1) και (2) βρίσκουμε $\boxed{\alpha = -7 \text{ και } \beta = -8}$

5.13 3)

Πρέπει

$$\boxed{\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x)} \Rightarrow \lim_{x \rightarrow -2^-} (\alpha x^2 + 3\beta x - 2) = \lim_{x \rightarrow -2^+} (\beta x + \alpha) \Rightarrow$$

$$\Rightarrow \alpha(-2)^2 + 3\beta(-2) - 2 = \beta(-2) + \alpha \Rightarrow 4\alpha - 6\beta - 2 = -2\beta + \alpha \Rightarrow 3\alpha - 4\beta = 2 \quad (1)$$

και

$$\boxed{\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)} \Rightarrow \lim_{x \rightarrow 1^-} (\beta x + \alpha) = \lim_{x \rightarrow 1^+} (2\alpha x - \beta) \Rightarrow$$

$$\Rightarrow \beta \cdot 1 + \alpha = 2\alpha \cdot 1 - \beta \Rightarrow -\alpha + 2\beta = 0 \quad (2)$$

Λύνοντας το σύστημα των (1) και (2) βρίσκουμε $\boxed{\alpha = 2 \text{ και } \beta = 1}$

5.13 4)

Πρέπει

$$\boxed{\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)} \Rightarrow \lim_{x \rightarrow 0^-} (2\alpha x + 2\beta) = \lim_{x \rightarrow 0^+} (3x + 2) \Rightarrow$$

$$\Rightarrow 2\alpha \cdot 0 + 2\beta = 3 \cdot 0 + 2 \Rightarrow 2\beta = 2 \Rightarrow \boxed{\beta = 1}$$

και

$$\boxed{\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x)} \Rightarrow \lim_{x \rightarrow 5^-} (3x + 2) = \lim_{x \rightarrow 5^+} (x^2 + \alpha x + 2\beta) \Rightarrow$$

$$\Rightarrow 3 \cdot 5 + 2 = 5^2 + \alpha \cdot 5 + 2\beta \Rightarrow 17 = 25 + 5\alpha + 2\beta \xrightarrow{\beta=1} -10 = 5\alpha \Rightarrow \boxed{\alpha = -2}$$

Πρέπει

$$\boxed{\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)} \Rightarrow \lim_{x \rightarrow 1^-} (3\alpha x^2 + 2\beta x + 1) = \lim_{x \rightarrow 1^+} (x^2 + \alpha x + \beta) \Rightarrow$$

$$\Rightarrow 3\alpha \cdot 1^2 + 2\beta \cdot 1 + 1 = 1^2 + \alpha \cdot 1 + \beta \Rightarrow 3\alpha + 2\beta + 1 = 1 + \alpha + \beta \Rightarrow 2\alpha + \beta = 0 \quad (1)$$

και

$$\boxed{\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)} \Rightarrow \lim_{x \rightarrow 2^-} (x^2 + \alpha x + \beta) = \lim_{x \rightarrow 2^+} (\alpha x^2 - 2\beta x - 2) \Rightarrow$$

$$\Rightarrow 2^2 + \alpha \cdot 2 + \beta = \alpha \cdot 2^2 - 2\beta \cdot 2 - 2 \Rightarrow 4 + 2\alpha + \beta = 4\alpha - 4\beta - 2 \Rightarrow -2\alpha + 5\beta = -6 \quad (2)$$

Λύνοντας το σύστημα των (1) και (2) βρίσκουμε $\boxed{\alpha = \frac{1}{2} \text{ και } \beta = -1}$