α) $f(x)-f(y)=f\left(\frac{x}{y}\right) \overset{\text{θέτουμε } x=y}{\Rightarrow} \cancel{f(y)}-\cancel{f(y)}=f\left(\frac{y}{y}\right)\Rightarrow \boxed{0=f(1)}$

β) i) Αν $\boxed{f(x_1)=f(x_2)}\Rightarrow f(x_1)-f(x_2)=0 \overset{f(x)-f(y)=f\left(\frac{x}{y}\right)}{\Rightarrow} f\left(\frac{x_1}{x_2}\right)=0 \overset{\text{μοναδική ρίζα το 1}}{\Rightarrow}$

$\frac{x_1}{x_2}=1\Rightarrow \boxed{x_1=x_2}$

Άρα η f είναι 1 – 1

ii) $f(x)+f\left(x^3-x+3\right)=f\left(x^2+2\right)+f(2x-1)\Rightarrow$

$\Rightarrow f\left(x^3-x+3\right)-f\left(x^2+2\right)=f(2x-1)-f(x) \overset{f(x)-f(y)=f\left(\frac{x}{y}\right)}{\Rightarrow}$

$\Rightarrow f\left(\frac{x^3-x+3}{x^2+2}\right)=f\left(\frac{2x-1}{x}\right)\overset{f:1-1}{\Rightarrow}\frac{x^3-x+3}{x^2+2}=\frac{2x-1}{x} \overset{\text{περιορισμός: } x\neq 0}{\Rightarrow}$

$\Rightarrow \left(x^3-x+3\right)x=(2x-1)\left(x^2+2\right)\Rightarrow x^4-\cancel{x^2}+3x=2x^3+4x-\cancel{x^2}-2\Rightarrow$

$x^4-2x^3-x+2=0\Rightarrow x^3(x-2)-(x-2)=0\Rightarrow (x-2)\left(x^3-1\right)=0\Rightarrow$

$\Rightarrow x-2=0 \text{ ή } x^3=1\Rightarrow \boxed{x=2 \text{ ή } x=1}$