

Έχουμε ότι

$$f(x^2) - f^2(x) \geq \frac{1}{4} \Rightarrow 4f(x^2) - 4f^2(x) \geq 1 \Rightarrow 4f(x^2) - 4f^2(x) - 1 \geq 0$$

Οπότε

$$4f(x^2) - 4f^2(x) - 1 \geq 0 \xrightarrow{\text{θέτουμε } x=0} 4f(0^2) - 4f^2(0) - 1 \geq 0 \Rightarrow [2f(0) - 1]^2 = 0 \Rightarrow$$

$$\Rightarrow 2f(0) - 1 = 0 \Rightarrow 2f(0) = 1 \Rightarrow f(0) = \frac{1}{2} \quad (1)$$

Ακόμη

$$4f(x^2) - 4f^2(x) - 1 \geq 0 \xrightarrow{\text{θέτουμε } x=1} 4f(1^2) - 4f^2(1) - 1 \geq 0 \Rightarrow [2f(1) - 1]^2 = 0 \Rightarrow$$

$$\Rightarrow 2f(1) - 1 = 0 \Rightarrow 2f(1) = 1 \Rightarrow f(1) = \frac{1}{2} \quad (2)$$

Επομένως $(1), (2) \Rightarrow f(0) = f(1)$ και άρα η f δεν είναι 1-1