

$$\alpha) \quad \text{Av} \quad \boxed{f(x_1) = f(x_2)} \quad \overset{f(x) = x - \ln f(x)}{\Rightarrow} \quad x_1 + \ln f(x_1) = x_2 + \ln f(x_2) \quad \overset{f(x_1) = f(x_2) \Rightarrow \ln f(x_1) = \ln f(x_2)}{\Rightarrow} \quad \boxed{x_1 = x_2}$$

$$\beta) \quad f(x) = x - \ln f(x) \quad \overset{\text{θέτουμε όπου } x \text{ το } f^{-1}(x)}{\Rightarrow} \quad f(f^{-1}(x)) = f^{-1}(x) - \ln f(f^{-1}(x)) \quad \overset{f(f^{-1}(x)) = x}{\Rightarrow} \\ \Rightarrow x = f^{-1}(x) - \ln x \Rightarrow \boxed{f^{-1}(x) = \ln x + x}$$