

22.4 1)

$$\begin{aligned} f'(x) = 2f(x) &\Rightarrow f'(x) - 2f(x) = 0 \quad \begin{array}{c} \text{πολλαπλασιάζουμε} \\ \text{με } e^{-2x} \end{array} \Rightarrow f'(x)e^{-2x} - 2f(x)e^{-2x} = 0 \Rightarrow \\ &\Rightarrow [f(x)e^{-2x}]' = 0 \Rightarrow f(x)e^{-2x} = c \Rightarrow f(x) = ce^{2x}, \text{ για κάθε } x \in \mathbb{R} \end{aligned}$$

22.4 2)

$$\begin{aligned} f'(x) = -f(x) &\Rightarrow f'(x) + f(x) = 0 \quad \begin{array}{c} \text{πολλαπλασιάζουμε} \\ \text{με } e^x \end{array} \Rightarrow f'(x)e^x + f(x)e^x = 0 \Rightarrow \\ &\Rightarrow [f(x)e^x]' = 0 \Rightarrow f(x)e^x = c \Rightarrow f(x) = ce^{-x}, \text{ για κάθε } x \in \mathbb{R} \end{aligned}$$

22.4 3)

$$\begin{aligned} f'(x) = 3f(x) &\Rightarrow f'(x) - 3f(x) = 0 \quad \begin{array}{c} \text{πολλαπλασιάζουμε} \\ \text{με } e^{-3x} \end{array} \Rightarrow f'(x)e^{-3x} - 3f(x)e^{-3x} = 0 \Rightarrow \\ &\Rightarrow [f(x)e^{-3x}]' = 0 \Rightarrow f(x)e^{-3x} = c \Rightarrow f(x) = ce^{3x}, \text{ για κάθε } x \in \mathbb{R} \end{aligned}$$