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$$g'(x+x^3)=8x \Rightarrow g'(x+x^3) \cdot (1+3x^2)=8x \cdot (1+3x^2) \Rightarrow [g'(x+x^3)]' = 24x^3+8x$$

$$\Rightarrow [g'(x+x^3)]' = (6x^4+4x^2)' \Rightarrow g(x+x^3)=6x^4+4x^2+c \quad (1)$$

Ακόμη

$$(1) \xrightarrow{x=1} g(1+1^3)=6 \cdot 1^4+4 \cdot 1^2+c \Rightarrow g(2)=10+c \xrightarrow{g(2)=10} 10=10+c \Rightarrow c=0 \quad (2)$$

$$(1), (2) \Rightarrow g(x+x^3)=6x^4+4x^2 \xrightarrow{x=0} g(0+0^3)=6 \cdot 0^4+4 \cdot 0^2 \Rightarrow \boxed{g(0)=0}$$