22.28

$$f''(x)+f'(x)=2e^x \overset{\text{πολλαπλασιάζουμε με } e^x}{\Rightarrow} f''(x)e^x+f'(x)e^x=2e^{2x} \Rightarrow$$

$$\Rightarrow \left[f'(x)e^x\right]'=\left(e^{2x}\right)' \Rightarrow f'(x)e^x=e^{2x}+c \overset{\text{πολλαπλασιάζουμε με } e^{-x}}{\Rightarrow} f'(x)=e^x+ce^{-x} \quad (1)$$

$$(1) \overset{x=0}{\Rightarrow} f'(0)=e^0+ce^0 \overset{f'(0)=0}{\Rightarrow} c=-1 \quad (2)$$

$$(1),(2) \Rightarrow f'(x)=e^x-e^{-x} \Rightarrow f'(x)=\left(e^x+e^{-x}\right)' \Rightarrow f(x)=e^x+e^{-x}+c_1 \quad (3)$$

$$(3) \overset{x=0}{\Rightarrow} f(0)=e^0+e^0+c_1 \overset{f(0)=0}{\Rightarrow} c_1=-2$$

$$(3),(4) \Rightarrow \boxed{f(x)=e^x+e^{-x}-2}$$