

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

22.26 1)

$$xf'(x) + f(x) = 0 \Rightarrow [xf(x)]' = 0 \Rightarrow xf(x) = c \stackrel{x \neq 0}{\Rightarrow} f(x) = \frac{c}{x} \quad (1)$$

Ακόμη

$$f(2) = 6 \stackrel{(1)}{\Rightarrow} \frac{c}{2} = 6 \Rightarrow c = 12 \quad (2)$$

$$(1), (2) \Rightarrow \boxed{f(x) = \frac{12}{x}}$$

22.26 2)

$$xf'(x) + f(x) = e^x \Rightarrow [xf(x)]' = (e^x)' \Rightarrow xf(x) = e^x + c \stackrel{x \neq 0}{\Rightarrow} f(x) = \frac{e^x + c}{x} \quad (1)$$

Ακόμη

$$f(1) = e \stackrel{(1)}{\Rightarrow} \frac{e^1 + c}{1} = e \Rightarrow c = 0 \quad (2)$$

$$(1), (2) \Rightarrow \boxed{f(x) = \frac{e^x}{x}}$$

22.26 3)

$$e^{f(x)} = \frac{1}{f'(x)} \Rightarrow e^{f(x)} f'(x) = 1 \Rightarrow [e^{f(x)}]' = (x)' \Rightarrow e^{f(x)} = x + c \quad (1)$$

Ακόμη

$$(1) \stackrel{x=1}{\Rightarrow} e^{f(1)} = 1 + c \stackrel{f(1)=0}{\Rightarrow} c = 0 \quad (2)$$

$$(1), (2) \Rightarrow e^{f(x)} = x \Rightarrow \boxed{f(x) = \ln x}$$

22.26 4)

$$f'(x) = f(x) \ln f(x) \stackrel{\text{διαφορούμε με } f(x) \neq 0}{\Rightarrow} \frac{f'(x)}{f(x)} = \ln f(x) \Rightarrow [\ln f(x)]' = \ln f(x) \Rightarrow$$

$$\Rightarrow \ln f(x) = ce^x \quad (1)$$

Ακόμη

$$(1) \stackrel{x=0}{\Rightarrow} \ln f(0) = ce^0 \stackrel{f(0)=e^2}{\Rightarrow} \ln e^2 = c \Rightarrow c = 2 \quad (2)$$

$$(1), (2) \Rightarrow \ln f(x) = 2e^x \Rightarrow \boxed{f(x) = e^{2e^x}}$$

22.26 5)

$$x^2 e^{f(x)} f'(x) = x \sin x - \eta \mu x \stackrel{\text{διαφορούμε με } x^2 \neq 0}{\Rightarrow} e^{f(x)} f'(x) = \frac{x \sin x - \eta \mu x}{x^2} \Rightarrow$$

$$\Rightarrow [e^{f(x)}]' = \left(\frac{\eta \mu x}{x} \right)' \Rightarrow e^{f(x)} = \frac{\eta \mu x}{x} + c \Rightarrow f(x) = \ln \left(\frac{\eta \mu x}{x} + c \right) \quad (1)$$

Ακόμη

$$f\left(\frac{\pi}{2}\right)=\ln\frac{2}{\pi}\overset{(1)}{\Rightarrow}\ln\left(\frac{\eta\mu\frac{\pi}{2}}{\frac{\pi}{2}}+c\right)=\ln\frac{2}{\pi}\Rightarrow\frac{1}{\frac{\pi}{2}}+c=\frac{2}{\pi}\Rightarrow\frac{2}{\pi}+c=\frac{2}{\pi}\Rightarrow c=0 \quad (2)$$

$$(1),(2)\Rightarrow \boxed{f\left(x\right)=\ln\left(\frac{\eta\mu x}{x}\right)}$$