

22.18 1)

$$xf'(x) = (x-1)f(x) \Rightarrow xf'(x) = xf(x) - f(x) \Rightarrow xf'(x) + f(x) = xf(x) \Rightarrow$$

$$\Rightarrow [xf(x)]' = xf(x) \Rightarrow xf(x) = ce^x \Rightarrow f(x) = \frac{ce^x}{x} \quad (1)$$

$$\text{Ακόμη } f(2) = 3e^2 \Rightarrow \frac{ce^2}{2} = 3e^2 \Rightarrow ce^2 = 6e^2 \Rightarrow c = 6 \quad (2)$$

$$\text{Επομένως } (1), (2) \Rightarrow f(x) = \frac{6e^x}{x}$$

22.18 2)

$$x^2f'(x) - 2xf(x) = x^2f(x) \xrightarrow{\text{διαιρούμε με } x^4 \neq 0} \frac{x^2f'(x) - 2xf(x)}{x^4} = \frac{x^2f(x)}{x^4} \Rightarrow$$

$$\Rightarrow \left[\frac{f(x)}{x^2} \right]' = \frac{f(x)}{x^2} \Rightarrow \frac{f(x)}{x^2} = ce^x \Rightarrow f(x) = cx^2e^x \quad (1)$$

$$\text{Ακόμη } f(1) = 2e \Rightarrow c \cdot 1^2 e^1 = 2e \Rightarrow ce = 2e \Rightarrow c = 2 \quad (2)$$

$$\text{Επομένως } (1), (2) \Rightarrow f(x) = 2x^2e^x$$

22.18 3)

$$f'(x)\eta\mu x = (\eta\mu x - \sigma\upsilon\nu x)f(x) \Rightarrow f'(x)\eta\mu x = f(x)\eta\mu x - f(x)\sigma\upsilon\nu x \Rightarrow$$

$$\Rightarrow f'(x)\eta\mu x + f(x)\sigma\upsilon\nu x = f(x)\eta\mu x \Rightarrow [f(x)\eta\mu x]' = f(x)\eta\mu x \Rightarrow$$

$$\Rightarrow f(x)\eta\mu x = ce^x \xrightarrow{x \in (0, \pi) \Rightarrow \eta\mu x \neq 0} f(x) = \frac{ce^x}{\eta\mu x} \quad (1)$$

$$\text{Ακόμη } f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \Rightarrow \frac{ce^{\frac{\pi}{2}}}{\eta\mu \frac{\pi}{2}} = e^{\frac{\pi}{2}} \Rightarrow ce^{\frac{\pi}{2}} = e^{\frac{\pi}{2}} \Rightarrow c = 1 \quad (2)$$

$$\text{Επομένως } (1), (2) \Rightarrow f(x) = \frac{e^x}{\eta\mu x}$$