

22.15 1)

Είναι

$$\left. \begin{aligned} \left(\frac{f(x)}{x^2+1} \right)' &= \frac{f(x)}{x^2+1} \\ \text{για κάθε } x \in \mathbb{R} \end{aligned} \right\} \Rightarrow \frac{f(x)}{x^2+1} = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη (1)} \stackrel{\text{θέτουμε } x=1}{\Rightarrow} \frac{f(1)}{1^2+1} = ce^1 \stackrel{f(1)=6e}{\Rightarrow} \frac{6e}{2} = ce \Rightarrow c = 3 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow \frac{f(x)}{x^2+1} = 3e^x \Rightarrow f(x) = 3e^x (x^2+1)$$

22.15 2)

Είναι

$$\left. \begin{aligned} f'(x) &= f(x) \\ \text{για κάθε } x \in \mathbb{R} \end{aligned} \right\} \Rightarrow f(x) = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη } f(2) = -3e^2 \stackrel{f(x)=ce^x}{\Rightarrow} ce^2 = -3e^2 \Rightarrow c = -3 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow f(x) = -3e^x$$

22.15 3)

Είναι

$$\left. \begin{aligned} (f(x)+x)' &= f(x)+x \\ \text{για κάθε } x \in \mathbb{R} \end{aligned} \right\} \Rightarrow f(x)+x = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη (1)} \stackrel{\text{θέτουμε } x=1}{\Rightarrow} f(1)+1 = ce^1 \stackrel{f(1)=2e-1}{\Rightarrow} 2e-1+1 = ce \Rightarrow 2e = ce \Rightarrow c = 2 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow f(x)+x = 2e^x \Rightarrow f(x) = 2e^x - x$$

22.15 4)

Είναι

$$\left. \begin{aligned} (2f(x)-3x^5)' &= 2f(x)-3x^5 \\ \text{για κάθε } x \in \mathbb{R} \end{aligned} \right\} \Rightarrow 2f(x)-3x^5 = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη (1)} \stackrel{\text{θέτουμε } x=0}{\Rightarrow} 2f(0)-3 \cdot 0^5 = ce^0 \stackrel{f(0)=3}{\Rightarrow} 2 \cdot 3 - 3 \cdot 0^5 = ce^0 \Rightarrow c = 6 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow 2f(x)-3x^5 = 6e^x \Rightarrow 2f(x) = 3x^5 + 6e^x \Rightarrow f(x) = \frac{3x^5 + 6e^x}{2}$$

22.15 5)

Είναι

$$\left. \left(\frac{f(x)}{\sin x + 2} \right)' = \frac{f(x)}{\sin x + 2} \right\} \Rightarrow \frac{f(x)}{\sin x + 2} = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη (1)} \Rightarrow \frac{f\left(\frac{\pi}{2}\right)}{\sin \frac{\pi}{2} + 2} = ce^{\frac{\pi}{2}} \Rightarrow \frac{f\left(\frac{\pi}{2}\right) = 8e^{\frac{\pi}{2}}}{0 + 2} = ce^{\frac{\pi}{2}} \Rightarrow 4e^{\frac{\pi}{2}} = ce^{\frac{\pi}{2}} \Rightarrow c = 4 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow \frac{f(x)}{\sin x + 2} = 4e^x \Rightarrow f(x) = 4e^x (\sin x + 2)$$

22.15 6)

Είναι

$$\left. \left(f(x) \sqrt{x^2 + 4} \right)' = f(x) \sqrt{x^2 + 4} \right\} \Rightarrow f(x) \sqrt{x^2 + 4} = ce^x \quad (1) \quad , \quad \text{για κάθε } x \in \mathbb{R}$$

$$\text{Ακόμη (1)} \Rightarrow \frac{f(0)}{\sqrt{0^2 + 4}} = ce^0 \Rightarrow \frac{f(0) = 4}{4} = c \Rightarrow c = 1 \quad (2)$$

$$\text{Επομένως (1),(2)} \Rightarrow f(x) \sqrt{x^2 + 4} = e^x \Rightarrow f(x) = \frac{e^x}{\sqrt{x^2 + 4}}$$