Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

22.14 1)

$$f'(x)=2xe^{-f(x)}\Rightarrow f'(x)=\frac{2x}{e^{f(x)}}\Rightarrow e^{f(x)}f'(x)=2x\Rightarrow\left[e^{f(x)}\right]'=\left(x^2\right)'\Rightarrow$$

$$\Rightarrow e^{f(x)}=x^2+c\Rightarrow f(x)=\ln\left(x^2+c\right)\quad(1)$$

Ακόμη $f(0)=0\overset{(1)}{\Rightarrow}\ln\left(0^2+c\right)=0\Rightarrow\ln c=0\Rightarrow c=1\quad(2)$

Επομένως $(1),(2)\Rightarrow f(x)=\ln\left(x^2+1\right)$

22.14 2)

$$\eta\mu^2x\cdot f'(x)=f^2(x)\overset{\substack{f(x)\neq0\\ x\in\left(0,\frac{\pi}{2}\right)\Rightarrow\eta\mu^2x\neq0}}{\Rightarrow}\frac{f'(x)}{f^2(x)}=\frac{1}{\eta\mu^2x}\Rightarrow-\frac{f'(x)}{f^2(x)}=-\frac{1}{\eta\mu^2x}\Rightarrow$$

$$\Rightarrow\left[\frac{1}{f(x)}\right]'=(\sigma\varphi x)'\Rightarrow\frac{1}{f(x)}=\sigma\varphi x+c\Rightarrow f(x)=\frac{1}{\sigma\varphi x+c}\quad(1)$$

Ακόμη

$$(1)\overset{x=\frac{\pi}{4}}{\Rightarrow}f\left(\frac{\pi}{4}\right)=\frac{1}{\sigma\varphi\frac{\pi}{4}+c}\overset{f\left(\frac{\pi}{4}\right)=1}{\Rightarrow}1=\frac{1}{1+c}\Rightarrow1+c=1\Rightarrow c=0\quad(2)$$

$$(1),(2)\Rightarrow\boxed{f(x)=\varepsilon\varphi x}$$

22.14 3)

$$f'(x)\sqrt{x^2+1}=xe^{-f(x)}\overset{\text{πολλαπλασιάζουμε με }\frac{e^{f(x)}}{\sqrt{x^2+1}}}{\Rightarrow}f'(x)e^{f(x)}=\frac{x}{\sqrt{x^2+1}}\Rightarrow$$

$$\Rightarrow\left[e^{f(x)}\right]'=\left(\sqrt{x^2+1}\right)'\Rightarrow e^{f(x)}=\sqrt{x^2+1}+c\Rightarrow f(x)=\ln\left(\sqrt{x^2+1}+c\right)\quad(1)$$

Ακόμη , επειδή η γραφική παράσταση της f διέρχεται από την αρχή των αξόνων , θα είναι

$$f(0)=0\overset{(1)}{\Rightarrow}\ln\left(\sqrt{0^2+1}+c\right)=0\Rightarrow\ln(1+c)=\ln1\Rightarrow\cancel{1}+c=\cancel{1}\Rightarrow c=0\quad(2)$$

$$(1),(2)\Rightarrow f(x)=\ln\sqrt{x^2+1}\Rightarrow f(x)=\ln\left(x^2+1\right)^{\frac{1}{2}}\Rightarrow\boxed{f(x)=\frac{\ln\left(x^2+1\right)}{2}}$$