

22.13 1)

Παρατηρούμε ότι

$$(x^2 + 1)f'(x) + 2xf(x) = 3x^2 \Rightarrow [(x^2 + 1)f(x)]' = (x^3)' \Rightarrow (x^2 + 1)f(x) = x^3 + c \quad (1)$$

Ακόμη

$$(x^2 + 1)f(x) = x^3 + c \xrightarrow{\text{θέτουμε } x=0} (0^2 + 1)f(0) = 0^3 + c \xrightarrow{f(0)=1} c = 1 \quad (2)$$

$$(1), (2) \Rightarrow (x^2 + 1)f(x) = x^3 + 1 \Rightarrow f(x) = \frac{x^3 + 1}{x^2 + 1}$$

22.13 2)

Παρατηρούμε ότι

$$xf'(x) + f(x) = \sin x \Rightarrow [xf(x)]' = (\eta\mu x)' \Rightarrow xf(x) = \eta\mu x + c \quad (1)$$

Ακόμη

$$xf(x) = \eta\mu x + c \xrightarrow{\text{θέτουμε } x=\pi} \pi f(\pi) = \eta\mu\pi + c \xrightarrow{f(\pi)=0} c = 0 \quad (2)$$

$$(1), (2) \Rightarrow xf(x) = \eta\mu x \xrightarrow{x \neq 0} f(x) = \frac{\eta\mu x}{x}$$

22.13 3)

Παρατηρούμε ότι

$$f'(x)\sin x - f(x)\eta\mu x = 1 \Rightarrow [f(x)\sin x]' = (x)' \Rightarrow f(x)\sin x = x + c \quad (1)$$

Ακόμη

$$f(x)\sin x = x + c \xrightarrow{\text{θέτουμε } x=0} f(0)\sin 0 = 0 + c \xrightarrow{f(0)=1} c = 1 \quad (2)$$

$$(1), (2) \Rightarrow f(x)\sin x = x + 1 \xrightarrow{x \in [0, \frac{\pi}{2}] \Rightarrow \sin x \neq 0} f(x) = \frac{x + 1}{\sin x}$$

22.13 4)

Παρατηρούμε ότι

$$xf'(x) - f(x) = x^2 \xrightarrow{\text{διαιρούμε με } x \neq 0} \frac{xf'(x) - f(x)}{x^2} = 1 \Rightarrow \left(\frac{f(x)}{x} \right)' = (x)' \Rightarrow \frac{f(x)}{x} = x + c \quad (1)$$

Ακόμη

$$\frac{f(x)}{x} = x + c \xrightarrow{\text{θέτουμε } x=1} \frac{f(1)}{1} = 1 + c \xrightarrow{f(1)=3} c = 2 \quad (2)$$

$$(1), (2) \Rightarrow \frac{f(x)}{x} = x + 2 \Rightarrow f(x) = x^2 + 2x$$