Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

22.10 1)

Έχουμε ότι

$$f'(x)=e^x+xe^x \Rightarrow f'(x)=\left(xe^x\right)' \Rightarrow f(x)=xe^x+c \quad (1)$$

Ακόμη

$$f(1)=e-2 \overset{(1)}{\Rightarrow} 1\cdot e^1+c=e-2 \Rightarrow \cancel{e}+c=\cancel{e}-2 \Rightarrow c=-2 \quad (2)$$

$$(1),(2) \Rightarrow \boxed{f(x)=xe^x-2}$$

22.10 2)

Έχουμε ότι

$$f'(x)=\frac{1}{2\sqrt{x}}+2x-3 \Rightarrow f'(x)=\left(\sqrt{x}+x^2-3x\right)' \Rightarrow f(x)=\sqrt{x}+x^2-3x+c \quad (1)$$

Ακόμη

$$f(4)=10 \overset{(1)}{\Rightarrow} \sqrt{4}+4^2-3\cdot 4+c=10 \Rightarrow 2+16-12+c=10 \Rightarrow c=4 \quad (2)$$

$$(1),(2) \Rightarrow \boxed{f(x)=\sqrt{x}+x^2-3x+4}$$

22.10 3)

Έχουμε ότι

$$f'(x)=\eta\mu x+x\sigma\upsilon\nu x \Rightarrow f'(x)=\left(x\eta\mu x\right)' \Rightarrow f(x)=x\eta\mu x+c \quad (1)$$

Ακόμη

$$f(\pi)=-2 \overset{(1)}{\Rightarrow} \pi\eta\mu\pi+c=-2 \Rightarrow c=-2 \quad (2)$$

$$(1),(2) \Rightarrow \boxed{f(x)=x\eta\mu x-2}$$

22.10 4)

Έχουμε ότι

$$f'(x)=\frac{x}{\sqrt{x^2+1}} \Rightarrow f'(x)=\left(\sqrt{x^2+1}\right)' \Rightarrow f(x)=\sqrt{x^2+1}+c \quad (1)$$

Ακόμη

$$f\left(2\sqrt{2}\right)=2f(0) \overset{(1)}{\Rightarrow} \sqrt{\left(2\sqrt{2}\right)^2+1}+c=2\left(\sqrt{0^2+1}+c\right) \Rightarrow \sqrt{8+1}+c=2(1+c) \Rightarrow$$

$$\Rightarrow 3+c=2+2c \Rightarrow c=1 \quad (2)$$

$$(1),(2) \Rightarrow \boxed{f(x)=\sqrt{x^2+1}+1}$$

22.10 5)

Έχουμε ότι

$$f'(x)=e^x(\sigma\upsilon\nu x-\eta\mu x) \Rightarrow f'(x)=\left(e^x\sigma\upsilon\nu x\right)' \Rightarrow f(x)=e^x\sigma\upsilon\nu x+c \quad (1)$$

Ακόμη

$$f\left(\frac{\pi}{2}\right)=1\overset{(1)}{\Rightarrow}e^{\frac{\pi}{2}}\text{συν}\frac{\pi}{2}+c=1\Rightarrow c=1 \quad (2)$$

$$(1),(2)\Rightarrow \boxed{f(x)=e^{x}\text{συν}x+1}$$

22.10 6)

Έχουμε ότι

$$f'(x)=3\text{συν}3x+3x^2\Rightarrow f'(x)=\left(\text{ημ}3x+x^3\right)'\Rightarrow f(x)=\text{ημ}3x+x^3+c \quad (1)$$

Ακόμη

$$f(0)=1\overset{(1)}{\Rightarrow}\text{ημ}3\cdot 0+0^3+c=1\Rightarrow c=1 \quad (2)$$

$$(1),(2)\Rightarrow \boxed{f(x)=\text{ημ}3x+x^3+1}$$

22.10 7)

Έχουμε ότι

$$f'(x)=\frac{2x+1}{x^2+x+1}\Rightarrow f'(x)=\left(\ln\left(x^2+x+1\right)\right)'\Rightarrow f(x)=\ln\left(x^2+x+1\right)+c \quad (1)$$

Ακόμη

$$f(0)=4\overset{(1)}{\Rightarrow}f(x)=\ln\left(0^2+0+1\right)+c=4\Rightarrow c=4 \quad (2)$$

$$(1),(2)\Rightarrow \boxed{f(x)=\ln\left(x^2+x+1\right)+4}$$