2.9 1)

$$f(x+1)=2x^2-3x \overset{\text{θέτουμε } y=x+1\Rightarrow x=y-1}{\Rightarrow} f(y)=2(y-1)^2-3(y-1)\Rightarrow$$

$$\Rightarrow f(y)=2\left(y^2-2y+1\right)-3y+3\Rightarrow f(y)=2y^2-4y+2-3y+3\Rightarrow$$

$$\Rightarrow f(y)=2y^2-7y+5\Rightarrow \boxed{f(x)=2x^2-7x+5}$$

2.9 2)

$$f(x-2)=3x^2-1 \overset{\text{θέτουμε } y=x-2\Rightarrow x=y+2}{\Rightarrow} f(y)=3(y+2)^2-1\Rightarrow$$

$$\Rightarrow f(y)=3\left(y^2+4y+4\right)-1\Rightarrow f(y)=3y^2+12y+12-1\Rightarrow \boxed{f(y)=3y^2+12y+11}$$

2.9 3)

$$f(2x-3)=4x^2-10x+7 \overset{\text{θέτουμε } y=2x-3\Rightarrow x=\frac{y+3}{2}}{\Rightarrow} f(y)=4\left(\frac{y+3}{2}\right)^2-\cancel{10}^{5}\frac{y+3}{\cancel{2}}+7\Rightarrow$$

$$\Rightarrow f(y)=\cancel{4}\frac{y^2+6y+9}{\cancel{4}}-5y-15+7\Rightarrow f(y)=y^2+y+1\Rightarrow \boxed{f(x)=x^2+x+1}$$

2.9 4)

$$f\left(\frac{x^2}{x^2+1}\right)=\frac{x^4}{x^4+1} \overset{\text{θέτουμε } y=\frac{x^2}{x^2+1}\Rightarrow yx^2+y=x^2\Rightarrow yx^2-x^2=-y\Rightarrow x^2(y-1)=-y\Rightarrow x^2=-\frac{y}{y-1}\Rightarrow x^4=\left(\frac{y}{y-1}\right)^2}{\Rightarrow}$$

$$\Rightarrow f(y)=\frac{\left(\frac{y}{y-1}\right)^2}{\left(\frac{y}{y-1}\right)^2+1}\Rightarrow f(y)=\frac{\frac{y^2}{\cancel{(y-1)^2}}}{\frac{y^2+(y-1)^2}{\cancel{(y-1)^2}}}\Rightarrow$$

$$\Rightarrow f(y)=\frac{y^2}{2y^2-2y+1}\Rightarrow \boxed{f(x)=\frac{x^2}{2x^2-2x+1}}$$