

2.8 1)

$$(f \circ g)(x) = x^2 + 2x + 2 \Rightarrow f(g(x)) = x^2 + 2x + 2 \Rightarrow f(x+1) = x^2 + 2x + 2 \Rightarrow$$

θέτουμε $x+1=y \Rightarrow x=y-1$

$$\Rightarrow f(y) = (y-1)^2 + 2(y-1) + 2 \Rightarrow f(y) = y^2 - 2y + 1 + 2y - 2 + 2 \Rightarrow$$

$$\Rightarrow f(y) = y^2 - \cancel{2y} + 1 + \cancel{2y} - \cancel{2} + \cancel{2} \Rightarrow f(y) = y^2 + 1 \Rightarrow \boxed{f(x) = x^2 + 1}$$

2.8 2)

$$(f \circ g)(x) = x^2 - 4x + 1 \Rightarrow f(g(x)) = x^2 - 4x + 1 \Rightarrow f(3x+2) = x^2 - 4x + 1 \Rightarrow$$

θέτουμε $3x+2=y \Rightarrow x=\frac{y-2}{3}$

$$\Rightarrow f(y) = \left(\frac{y-2}{3}\right)^2 - 4\left(\frac{y-2}{3}\right) + 1 \Rightarrow f(y) = \frac{y^2 - 4y + 4}{9} - \frac{4y - 8}{3} + 1 \Rightarrow$$

$$\Rightarrow f(y) = \frac{y^2 - 4y + 4 - 12y + 24 + 9}{9} \Rightarrow \boxed{f(y) = \frac{y^2 - 16y + 37}{9}}$$

2.8 3)

$$(f \circ g)(x) = \sqrt{1+x^2} \Rightarrow f(g(x)) = \sqrt{1+x^2} \Rightarrow f(-x^2) = \sqrt{1+x^2} \Rightarrow$$

θέτουμε $-x^2=y \Rightarrow x^2=-y$

$$\Rightarrow f(y) = \sqrt{1-y} \Rightarrow \boxed{f(x) = \sqrt{1-x}}$$