18.8 1)

α) $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = \boxed{0}$

β) $\lim_{x\to -\infty} x^2 e^{2x} = \lim_{x\to -\infty}\frac{x^2}{\frac{1}{e^{2x}}} = \lim_{x\to -\infty}\frac{x^2}{e^{-2x}} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to -\infty}\frac{2x}{-2e^{-2x}} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to -\infty}\frac{1}{2e^{-2x}} = \frac{1}{+\infty} = \boxed{0}$

18.8 2)

$\lim_{x\to 0^+}(e^x-1)\ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{e^x-1}} \overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{e^x}{(e^x-1)^2}} = \lim_{x\to 0^+}\left[-\frac{(e^x-1)^2}{xe^x}\right] \overset{\frac{0}{0}}{\underset{DLH}{=}}$

$= \lim_{x\to 0^+}\left[-\frac{2e^x(e^x-1)}{e^x+xe^x}\right] = -\frac{2\cdot e^0(e^0-1)}{e^0+0\cdot e^0} = \boxed{0}$

18.8 3)

$\lim_{x\to 0^+} xe^{\frac{3}{x}} = \lim_{x\to 0^+}\frac{e^{\frac{3}{x}}}{\frac{1}{x}} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{-\frac{3}{x^2}e^{\frac{3}{x}}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 3e^{\frac{3}{x}} = 3\cdot(+\infty) = \boxed{+\infty}$

18.8 4)

$\lim_{x\to +\infty} x\left(e^{\frac{1}{x}}-1\right) = \lim_{x\to +\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to +\infty}\frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}} = \lim_{x\to +\infty} e^{\frac{1}{x}} = e^0 = \boxed{1}$

18.8 5)

$\lim_{x\to 0^+}\ln x\cdot\ln(x+1) = \lim_{x\to 0^+}\frac{\ln(x+1)}{\frac{1}{\ln x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{\frac{1}{x+1}}{\frac{1}{\ln^2 x}\cdot\frac{1}{x}} = \lim_{x\to 0^+}\frac{\frac{1}{x+1}}{\frac{1}{\ln^2 x}\cdot\frac{1}{x}} =$

$= \lim_{x\to 0^+}\frac{x\cdot\ln^2 x}{x+1} = \lim_{x\to 0^+}\frac{1}{x+1}\cdot\lim_{x\to 0^+}(x\ln^2 x) = \frac{1}{0+1}\cdot\lim_{x\to 0^+}\frac{\ln^2 x}{\frac{1}{x}} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{2\cdot\frac{1}{x}\ln x}{-\frac{1}{x^2}} =$

$= -2\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}} -2\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = -2\lim_{x\to 0^+}(-x) = \boxed{0}$

18.8 6)

$$\lim_{x\to+\infty} x\cdot \eta\mu\frac{1}{x}=\lim_{x\to+\infty}\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{-\frac{1}{x^2}\sigma\upsilon\nu\frac{1}{x}}{-\frac{1}{x^2}}=\sigma\upsilon\nu 0=\boxed{1}$$

18.8 7)

$$\lim_{x\to2}\left[(2-x)\varepsilon\varphi\frac{\pi x}{4}\right]=\lim_{x\to2}\left[(2-x)\frac{\eta\mu\frac{\pi x}{4}}{\sigma\upsilon\nu\frac{\pi x}{4}}\right]=\lim_{x\to2}\eta\mu\frac{\pi x}{4}\cdot\lim_{x\to2}\frac{2-x}{\sigma\upsilon\nu\frac{\pi x}{4}}=$$

$$=\eta\mu\frac{2\pi}{4}\cdot\lim_{x\to2}\frac{2-x}{\sigma\upsilon\nu\frac{\pi x}{4}}\overset{\frac{0}{0}}{\underset{DLH}{=}}1\cdot\lim_{x\to2}\frac{-1}{-\frac{\pi}{4}\eta\mu\frac{\pi x}{4}}=\frac{1}{\frac{\pi}{4}\eta\mu\frac{2\pi}{4}}=\frac{1}{\frac{\pi}{4}}=\boxed{\frac{4}{\pi}}$$

18.8 8)

$$\lim_{x\to\frac{\pi}{4}}\left[\varepsilon\varphi 2x(1-\varepsilon\varphi x)\right]=\lim_{x\to\frac{\pi}{4}}\left[\frac{\eta\mu 2x}{\sigma\upsilon\nu 2x}\left(1-\frac{\eta\mu x}{\sigma\upsilon\nu x}\right)\right]=\lim_{x\to\frac{\pi}{4}}\left(\frac{\eta\mu 2x}{\sigma\upsilon\nu 2x}\cdot\frac{\sigma\upsilon\nu x-\eta\mu x}{\sigma\upsilon\nu x}\right)=$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\eta\mu 2x}{\sigma\upsilon\nu x}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\sigma\upsilon\nu x-\eta\mu x}{\sigma\upsilon\nu 2x}=\frac{\eta\mu 2\frac{\pi}{4}}{\sigma\upsilon\nu\frac{\pi}{4}}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\sigma\upsilon\nu x-\eta\mu x}{\sigma\upsilon\nu 2x}=\frac{1}{\frac{\sqrt{2}}{2}}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\sigma\upsilon\nu x-\eta\mu x}{\sigma\upsilon\nu 2x}\overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$=\frac{2}{\sqrt{2}}\cdot\lim_{x\to\frac{\pi}{4}}\frac{-\eta\mu x-\sigma\upsilon\nu x}{-2\eta\mu 2x}=\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\cdot\frac{-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{-2\cdot1}=\sqrt{2}\cdot\frac{-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}}{-2}=\sqrt{2}\cdot\frac{\sqrt{2}}{2}=\boxed{1}$$

18.8 9)

$$\lim_{x\to+\infty}\left[x\cdot\ln\left(\frac{x+2}{x}\right)\right]\overset{\lim\limits_{x\to+\infty}\frac{x+2}{x}=1\Rightarrow\boxed{\lim\limits_{x\to+\infty}\ln\left(\frac{x+2}{x}\right)=0}}{=}\lim_{x\to+\infty}\frac{\ln\left(\frac{x+2}{x}\right)}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$=\lim_{x\to+\infty}\frac{\frac{1}{\frac{x+2}{\cancel{x}}}\cdot\frac{x-(x+2)}{x^{\cancel{2}}}}{-\frac{1}{x^2}}=\lim_{x\to+\infty}\frac{\frac{\cancel{x}-\cancel{x}-2}{\cancel{x}(x+2)}}{-\frac{1}{x^{\cancel{2}}}}=\lim_{x\to+\infty}\frac{2x}{x+2}=\lim_{x\to+\infty}\frac{2\cancel{x}}{\cancel{x}\left(1+\frac{2}{x}\right)}=\boxed{0}$$

18.8 10)

$$\lim_{x\to0^+}\left(\eta\mu 2x\cdot e^{\frac{1}{\eta\mu 2x}}\right)=\lim_{x\to0^+}\frac{e^{\frac{1}{\eta\mu 2x}}}{\frac{1}{\eta\mu 2x}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{\cancel{-\frac{1}{\eta\mu^2 2x}\cdot\sigma\upsilon\nu 2x\cdot2}\cdot e^{\frac{1}{\eta\mu 2x}}}{\cancel{-\frac{1}{\eta\mu^2 2x}\cdot\sigma\upsilon\nu 2x\cdot2}}=$$

$$=\lim_{x\to0^+}e^{\frac{1}{\eta\mu 2x}}=\boxed{+\infty}$$

18.8 11)

$$\lim_{x\to e^+}\left[(\ln x-1)\cdot\ln(x-e)\right]=\lim_{x\to e^+}\frac{\ln(x-e)}{\frac{1}{\ln x-1}}\overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to e^+}\frac{\frac{1}{x-e}}{\frac{1}{(\ln x-1)^2}\cdot\frac{1}{x}}=$$

$$= \lim_{x\to e^+} \frac{x(\ln x -1)^2}{x-e} = \lim_{x\to e^+} x \cdot \lim_{x\to e^+}\frac{(\ln x-1)^2}{x-e} = e\cdot \lim_{x\to e^+}\frac{(\ln x-1)^2}{x-e} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= e\cdot\lim_{x\to e^+} \frac{2(\ln x-1)\cdot\frac{1}{x}}{1} = \cancel{e}\cdot 2(\ln e -1)\cdot \frac{1}{\cancel{e}} == \boxed{0}$$

18.8 12)

$$\lim_{x\to 0^+}(\varepsilon\varphi x\cdot \ln x) = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{\varepsilon\varphi x}} \overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{\varepsilon\varphi^2 x}\cdot\frac{1}{\sigma\upsilon\nu^2 x}} = \lim_{x\to 0^+}\left(-\frac{\varepsilon\varphi^2 x\,\sigma\upsilon\nu^2 x}{x}\right) =$$

$$= \lim_{x\to 0^+}\left(-\frac{\frac{\eta\mu^2 x}{\cancel{\sigma\upsilon\nu^2 x}}\cancel{\sigma\upsilon\nu^2 x}}{x}\right) = \lim_{x\to 0^+}\left(-\frac{\eta\mu^2 x}{x}\right) \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0^+}\left(-\frac{2\cdot\eta\mu x\cdot\sigma\upsilon\nu x}{1}\right) = \boxed{0}$$

18.8 13)

$$\lim_{x\to+\infty}\left[x^2\cdot\ln\left(\sigma\upsilon\nu\frac{1}{x}\right)\right] = \lim_{x\to+\infty}\frac{\ln\left(\sigma\upsilon\nu\frac{1}{x}\right)}{\frac{1}{x^2}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to+\infty}\frac{\frac{1}{\sigma\upsilon\nu\frac{1}{x}}\cdot\left(-\eta\mu\frac{1}{x}\right)\cdot\frac{-1}{\cancel{x^2}}}{-\frac{1}{x^{\cancel{4}}}\cdot 2\cancel{x}} =$$

$$= \lim_{x\to+\infty}\frac{\frac{1}{\sigma\upsilon\nu\frac{1}{x}}\cdot\eta\mu\frac{1}{x}}{-\frac{2}{x}} = \lim_{x\to+\infty}\frac{1}{-2\sigma\upsilon\nu\frac{1}{x}}\cdot\lim_{x\to+\infty}\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}} \overset{\lim_{x\to+\infty}\sigma\upsilon\nu\frac{1}{x}=\sigma\upsilon\nu 0=1}{=} -\frac{1}{2}\lim_{x\to+\infty}\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= -\frac{1}{2}\lim_{x\to+\infty}\frac{-\frac{\cancel{1}}{x^2}\sigma\upsilon\nu\frac{1}{x}}{-\frac{\cancel{1}}{x^2}} = -\frac{1}{2}\sigma\upsilon\nu 0 = \boxed{-\frac{1}{2}}$$

18.8 14)

$$\lim_{x\to+\infty}\left(x^2\cdot\eta\mu\frac{1}{x}\right) = \lim_{x\to+\infty}\frac{\eta\mu\frac{1}{x}}{\frac{1}{x^2}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to+\infty}\frac{\frac{-1}{\cancel{x^2}}\sigma\upsilon\nu\frac{1}{x}}{-\frac{1}{x^{\cancel{4}}}\cdot 2\cancel{x}} = \lim_{x\to+\infty}\frac{-\sigma\upsilon\nu\frac{1}{x}}{-\frac{2}{x}} =$$

$$= \lim_{x\to+\infty}\left(x\sigma\upsilon\nu\frac{1}{x}\right) = (+\infty)\cdot 1 = \boxed{+\infty}$$

18.8 15)

$$\lim_{x\to 0}\left[\sigma\varphi 2x\cdot(1-\sigma\upsilon\nu 2x)\right] = \lim_{x\to 0}\left[\frac{\sigma\upsilon\nu 2x}{\eta\mu 2x}\cdot(1-\sigma\upsilon\nu 2x)\right] =$$

$$= \lim_{x\to 0}\sigma\upsilon\nu 2x\cdot\lim_{x\to 0}\frac{1-\sigma\upsilon\nu 2x}{\eta\mu 2x} = 1\cdot\lim_{x\to 0}\frac{1-\sigma\upsilon\nu 2x}{\eta\mu 2x} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{\cancel{2}\cdot\eta\mu 2x}{\cancel{2}\cdot\sigma\upsilon\nu 2x} = \frac{0}{1} = \boxed{0}$$

18.8 16)

$$\lim_{x\to\frac{\pi}{4}^-}\left[\left(\frac{\pi}{2}-2x\right)\cdot \varepsilon\varphi 2x\right]=\lim_{x\to\frac{\pi}{4}^-}\left[\left(\frac{\pi}{2}-2x\right)\cdot\frac{\eta\mu 2x}{\sigma\upsilon\nu 2x}\right]=\lim_{x\to\frac{\pi}{4}^-}(\eta\mu 2x)\cdot\lim_{x\to\frac{\pi}{4}^-}\frac{\frac{\pi}{2}-2x}{\sigma\upsilon\nu 2x}=$$

$$=\eta\mu\frac{\pi}{2}\cdot\lim_{x\to\frac{\pi}{4}^-}\frac{\frac{\pi}{2}-2x}{\sigma\upsilon\nu 2x}\overset{\frac{0}{0}}{\underset{DLH}{=}}1\cdot\lim_{x\to\frac{\pi}{4}^-}\frac{\cancel{-2}}{\cancel{-2}\,\eta\mu 2x}=\frac{1}{\eta\mu\frac{\pi}{2}}=\boxed{1}$$

18.8 17)

$$\lim_{x\to+\infty}\left[2x^3\cdot e^{-(x+3)}\right]=\lim_{x\to+\infty}\frac{2x^3}{e^{x+3}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{6x^2}{e^{x+3}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{12x}{e^{x+3}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{12}{e^{x+3}}=\frac{12}{+\infty}=\boxed{0}$$

18.8 18)

$$\lim_{x\to1^+}\left[\ln x\cdot\ln(x-1)\right]=\lim_{x\to1^+}\frac{\ln(x-1)}{\frac{1}{\ln x}}\overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to1^+}\frac{\frac{1}{x-1}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}}=\lim_{x\to1^+}\left(-\frac{x\cdot\ln^2 x}{x-1}\right)=$$

$$\lim_{x\to1^+}(-x)\cdot\lim_{x\to1^+}\frac{\ln^2 x}{x-1}=-1\cdot\lim_{x\to1^+}\frac{\ln^2 x}{x-1}\overset{\frac{0}{0}}{\underset{DLH}{=}}-1\cdot\lim_{x\to1^+}\frac{2\ln x\cdot\frac{1}{x}}{1}=-1\cdot\lim_{x\to1^+}\left(2\cdot\ln 1\cdot\frac{1}{1}\right)=\boxed{0}$$

18.8 19)

$$\lim_{x\to+\infty}\left[3x\cdot\ln\left(1+\frac{1}{3x}\right)\right]=\lim_{x\to+\infty}\frac{\ln\left(1+\frac{1}{3x}\right)}{\frac{1}{3x}}=\lim_{x\to+\infty}\frac{\cancel{-\frac{1}{9x^2}\cdot 3}\cdot\frac{1}{1+\frac{1}{3x}}}{\cancel{-\frac{1}{9x^2}\cdot 3}}=\frac{1}{1+0}=\boxed{1}$$