Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.5 1)

$$\boxed{\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\text{ημ}x}\right)} = \lim_{x\to 0}\frac{\text{ημ}x - x}{x\text{ημ}x} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{\text{συν}x-1}{\text{ημ}x + x\text{συν}x} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= \lim_{x\to 0}\frac{\text{ημ}x}{\text{συν}x+\text{συν}x - x\text{ημ}x} = \frac{\text{ημ}0}{\text{συν}0+\text{συν}0-0\text{ημ}0} = \frac{0}{2} = \boxed{0}$$

18.5 2)

$$\boxed{\lim_{x\to 0^+}\left(\frac{1}{\text{εφ}x}-\frac{1}{x}\right)} = \lim_{x\to 0^+}\left(\text{σφ}x-\frac{1}{x}\right) = \lim_{x\to 0^+}\left(\frac{\text{συν}x}{\text{ημ}x}-\frac{1}{x}\right) = \lim_{x\to 0^+}\frac{x\text{συν}x-\text{ημ}x}{x\text{ημ}x} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= \lim_{x\to 0^+}\frac{\cancel{\text{συν}x} - x\text{ημ}x - \cancel{\text{συν}x}}{\text{ημ}x + x\text{συν}x} = \lim_{x\to 0^+}\frac{-x\text{ημ}x}{\text{ημ}x + x\text{συν}x} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0^+}\frac{-\text{ημ}x - x\text{συν}x}{\text{συν}x+\text{συν}x - x\text{ημ}x} =$$

$$= \frac{-\text{ημ}0 - 0\cdot\text{συν}0}{\text{συν}0+\text{συν}0-0\cdot\text{ημ}0} = \frac{0}{2} = \boxed{0}$$

18.5 3)

$$\boxed{\lim_{x\to 0^+}\left(\text{σφ}x-\frac{1}{x^2}\right)} = \lim_{x\to 0^+}\left(\frac{\text{συν}x}{\text{ημ}x}-\frac{1}{x^2}\right) = \lim_{x\to 0^+}\frac{x^2\text{συν}x-\text{ημ}x}{x^2\text{ημ}x} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= \lim_{x\to 0^+}\frac{2x\text{συν}x - x^2\text{ημ}x - \text{συν}x}{2x\text{ημ}x + x^2\text{συν}x} = \lim_{x\to 0^+}\frac{2x\text{συν}x - x^2\text{ημ}x - \text{συν}x}{x^2\left(2\frac{\text{ημ}x}{x}+\text{συν}x\right)} =$$

$$= \lim_{x\to 0^+}\frac{2x\text{συν}x - x^2\text{ημ}x - \text{συν}x}{2\frac{\text{ημ}x}{x}+\text{συν}x}\cdot\lim_{x\to 0^+}\frac{1}{x^2} = \frac{0-0-1}{2\cdot 1+1}\cdot(+\infty) = -\frac{1}{3}(+\infty) = \boxed{-\infty}$$

18.5 4)

$$\boxed{\lim_{x\to 0}\left(\frac{1}{2x}-\frac{1}{e^{2x}-1}\right)} = \lim_{x\to 0}\frac{e^{2x}-1-2x}{2x\left(e^{2x}-1\right)} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{2e^{2x}-2}{2\left(e^{2x}-1\right)+4xe^{2x}} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= \lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4e^{2x}+8xe^{2x}} = \frac{4\cdot 1}{4\cdot 1 + 4\cdot 1 + 0} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

18.5 5)

$$\boxed{\lim_{x\to 1}\left(\frac{3}{\ln x}-\frac{3}{x-1}\right)} = \lim_{x\to 1}\frac{3x-3-3\ln x}{(x-1)\ln x} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 1}\frac{3-\frac{3}{x}}{\ln x + \frac{x-1}{x}} = \lim_{x\to 1}\frac{\frac{3x-3}{\cancel{x}}}{\frac{x\ln x + x - 1}{\cancel{x}}} =$$

$$= \lim_{x\to 1}\frac{3x-3}{x\ln x + x - 1} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 1}\frac{3}{\ln x + \cancel{x}\frac{1}{\cancel{x}} + 1} = \frac{3}{1+1} = \boxed{\frac{3}{2}}$$