

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.3 1)

(Δεν κάνουμε κατευθείαν DLH)

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{10^x - 5^x}{x \sin^2 x \sqrt{1-x^2}} &= \lim_{x \rightarrow 0} \frac{1}{\sin^2 x \sqrt{1-x^2}} \cdot \lim_{x \rightarrow 0} \frac{10^x - 5^x}{x} = \\ &= \frac{1}{\sin^2 0 \cdot \sqrt{1-0^2}} \cdot \lim_{x \rightarrow 0} \frac{10^x - 5^x}{x} = 1 \cdot \lim_{x \rightarrow 0} \frac{10^x - 5^x}{x} = \lim_{x \rightarrow 0} \frac{10^x - 5^x}{x} \stackrel{\frac{0}{0}}{=} \text{DLH} \\ &= \lim_{x \rightarrow 0} \frac{10^x \ln 10 - 5^x \ln 5}{1} = \ln 10 - \ln 5 = \ln \frac{10}{5} = \ln 2\end{aligned}$$

18.3 2)

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{(x^2 + x + 1)(3^x - 1)}{2^x \cdot \ln(x+1)} &= \lim_{x \rightarrow 0} \frac{x^2 + x + 1}{2^x} \cdot \lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(x+1)} = \\ &= \frac{0^2 + 0 + 1}{2^0} \cdot \lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(x+1)} = 1 \cdot \lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(x+1)} = \lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(x+1)} \stackrel{\frac{0}{0}}{=} \text{DLH} \\ &= \lim_{x \rightarrow 0} \frac{(3^x - 1)'}{[\ln(x+1)]'} = \lim_{x \rightarrow 0} \frac{3^x \ln 3}{\frac{1}{x+1}} = \frac{3^0 \ln 3}{\frac{1}{0+1}} = \boxed{\ln 3}\end{aligned}$$

18.3 3)

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{3 \sin^3 x \eta \mu x}{2^x (1 - e^x)} &= \lim_{x \rightarrow 0} \frac{3 \sin^3 x}{2^x} \cdot \lim_{x \rightarrow 0} \frac{\eta \mu x}{1 - e^x} = \frac{3 \sin^3 0}{2^0} \cdot \lim_{x \rightarrow 0} \frac{\eta \mu x}{1 - e^x} = \frac{3}{1} \cdot \lim_{x \rightarrow 0} \frac{\eta \mu x}{1 - e^x} = \\ &= 3 \cdot \lim_{x \rightarrow 0} \frac{\eta \mu x}{1 - e^x} \stackrel{\frac{0}{0}}{=} \text{DLH} 3 \cdot \lim_{x \rightarrow 0} \frac{(\eta \mu x)'}{(1 - e^x)'} = 3 \cdot \lim_{x \rightarrow 0} \frac{\sigma \upsilon \nu x}{-e^x} = 3 \cdot \frac{1}{-1} = \boxed{-3}\end{aligned}$$