Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.2 1)

α) $$\lim_{x\to0}\frac{e^x-e^{-x}-2x}{x-\etaμx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(e^x-e^{-x}-2x\right)'}{\left(x-\etaμx\right)'}=\lim_{x\to0}\frac{e^x+e^{-x}-2}{1-συνx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(e^x+e^{-x}-2\right)'}{\left(1-συνx\right)'}$$

$$=\lim_{x\to0}\frac{e^x-e^{-x}}{\etaμx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(e^x-e^{-x}\right)'}{\left(\etaμx\right)'}=\lim_{x\to0}\frac{e^x+e^{-x}}{συνx}=\frac{e^0+e^0}{συν0}=2$$

β) $$\lim_{x\to+\infty}\frac{\ln\left(1+e^x\right)}{x+1}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\left[\ln\left(1+e^x\right)\right]'}{\left(x+1\right)'}=\lim_{x\to+\infty}\frac{\frac{1}{1+e^x}\cdot e^x}{1}=\lim_{x\to+\infty}\frac{e^x}{1+e^x}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}$$

$$=\lim_{x\to+\infty}\frac{\left(e^x\right)'}{\left(1+e^x\right)'}=\lim_{x\to+\infty}\frac{e^x}{e^x}=1$$

18.2 2)

$$\lim_{x\to0}\frac{e^x-x-1}{x^3-2x^2}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(e^x-x-1\right)'}{\left(x^3-2x^2\right)'}=\lim_{x\to0}\frac{e^x-1}{3x^2-4x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(e^x-1\right)'}{\left(3x^2-4x\right)'}=\lim_{x\to0}\frac{e^x}{6x-4}=\frac{e^0}{-4}=-\frac{1}{4}$$

18.2 3)

$$\lim_{x\to0}\frac{x^2}{1-συνx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(x^2\right)'}{\left(1-συνx\right)'}=\lim_{x\to0}\frac{2x}{\etaμx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(2x\right)'}{\left(\etaμx\right)'}=\lim_{x\to0}\frac{2}{συνx}=2$$

18.2 4)

$$\boxed{\lim_{x\to0}\frac{x^3}{x-\etaμx}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(x^3\right)'}{\left(x-\etaμx\right)'}=\lim_{x\to0}\frac{3x^2}{1-συνx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{6x}{\etaμx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{6}{συνx}=\boxed{6}$$

18.2 5)

$$\lim_{x\to0}\frac{x-\etaμx}{x\etaμx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(x-\etaμx\right)'}{\left(x\etaμx\right)'}=\lim_{x\to0}\frac{1-συνx}{\etaμx+xσυνx}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\left(1-συνx\right)'}{\left(\etaμx+xσυνx\right)'}=$$

$$=\lim_{x\to0}\frac{\etaμx}{συνx+συνx-x\etaμx}=\frac{0}{2}=\boxed{0}$$

18.2 6)

$$\boxed{\lim_{x\to+\infty}\frac{\ln\left(e^{2x}+1\right)}{2x+1}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\left[\ln\left(e^{2x}+1\right)\right]'}{\left(2x+1\right)'}=\lim_{x\to+\infty}\frac{\frac{1}{e^{2x}+1}\cdot e^{2x}\cdot \cancel{2}}{\cancel{2}}=\lim_{x\to+\infty}\frac{e^{2x}}{e^{2x}+1}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}$$

$$=\lim_{x\to+\infty}\frac{\left(e^{2x}\right)'}{\left(e^{2x}+1\right)'}=\lim_{x\to+\infty}\frac{2e^{2x}}{2e^{2x}}=\boxed{1}$$

18.2 7)

β) $\lim_{x\to+\infty}\frac{x^2+x+1}{x\ln x}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{(x^2+x+1)'}{(x\ln x)'}=\lim_{x\to+\infty}\frac{2x+1}{\ln x+\cancel{x}\frac{1}{\cancel{x}}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{(2x+1)'}{(\ln x+1)'}=$

$=\lim_{x\to+\infty}\frac{2}{\frac{1}{x}}=\lim_{x\to+\infty}(2x)=+\infty$

18.2 8)

$$\boxed{\lim_{x\to0}\frac{x-\eta\mu x}{e^x-e^{\eta\mu x}}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{1-\sigma\upsilon\nu x}{e^x-e^{\eta\mu x}\cdot\sigma\upsilon\nu x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{\eta\mu x}{e^x-e^{\eta\mu x}\cdot\sigma\upsilon\nu^2x+e^{\eta\mu x}\cdot\eta\mu x}\overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$=\lim_{x\to0}\frac{\sigma\upsilon\nu x}{e^x-e^{\eta\mu x}\cdot\sigma\upsilon\nu^3x-e^{\eta\mu x}\cdot2\sigma\upsilon\nu x\cdot(-\eta\mu x)+e^{\eta\mu x}\cdot\eta\mu^2x+e^{\eta\mu x}\cdot\sigma\upsilon\nu x}$$

$$=\lim_{x\to0}\frac{1}{1-1-0+0+1}=\boxed{1}$$

18.2 9)

$$\lim_{x\to0^+}\frac{x}{\ln(1-\sigma\upsilon\nu x)}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{(x)'}{[\ln(1-\sigma\upsilon\nu x)]'}=\lim_{x\to0^+}\frac{1}{\frac{1}{1-\sigma\upsilon\nu x}\cdot\eta\mu x}=\lim_{x\to0^+}\frac{1-\sigma\upsilon\nu x}{\eta\mu x}=$$

$$=\lim_{x\to0^+}\frac{1-\sigma\upsilon\nu x}{\eta\mu x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{(1-\sigma\upsilon\nu x)'}{(\eta\mu x)'}=\lim_{x\to0^+}\frac{\eta\mu x}{\sigma\upsilon\nu x}=\frac{0}{1}=0$$