Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.1 1)

α) $$\lim_{x\to 2}\frac{\ln(x-1)}{x-2}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to 2}\frac{[\ln(x-1)]'}{(x-2)'}=\lim_{x\to 2}\frac{\frac{1}{x-1}}{1}=\lim_{x\to 2}\frac{1}{x-1}=\frac{1}{2-1}=1$$

β) $$\lim_{x\to+\infty}\frac{e^x}{x}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{(e^x)'}{(x)'}=\lim_{x\to+\infty}\frac{e^x}{1}=+\infty$$

γ) $$\lim_{x\to\frac{\pi}{2}^+}\frac{\sigma\upsilon\nu x}{1-\eta\mu x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to\frac{\pi}{2}^+}\frac{(\sigma\upsilon\nu x)'}{(1-\eta\mu x)'}=\lim_{x\to\frac{\pi}{2}^+}\frac{-\eta\mu x}{-\sigma\upsilon\nu x}=\lim_{x\to\frac{\pi}{2}^+}\frac{\eta\mu x}{\sigma\upsilon\nu x}=$$

$$=\lim_{x\to\frac{\pi}{2}^+}\eta\mu x\cdot\lim_{x\to\frac{\pi}{2}^+}\frac{1}{\sigma\upsilon\nu x}\overset{\sigma\upsilon\nu x<0}{=}1\cdot(-\infty)=-\infty$$

18.1 2)

$$\boxed{\lim_{x\to 1}\frac{1-x^5}{x^2-1}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to 1}\frac{(1-x^5)'}{(x^2-1)'}=\lim_{x\to 1}\frac{-5x^{\not 4\,3}}{2\not x}=\lim_{x\to 1}\frac{-5x^3}{2}=\boxed{-\frac{5}{2}}$$

18.1 3)

$$\boxed{\lim_{x\to 0}\frac{\eta\mu 2x}{3x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to 0}\frac{(\eta\mu 2x)'}{(3x)'}=\lim_{x\to 0}\frac{2\sigma\upsilon\nu 2x}{3}=\frac{2\cdot 1}{3}=\boxed{\frac{2}{3}}$$

18.1 4)

$$\boxed{\lim_{x\to\frac{\pi}{2}}\frac{2\sigma\upsilon\nu x}{2x-\pi}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to\frac{\pi}{2}}\frac{(2\sigma\upsilon\nu x)'}{(2x-\pi)'}=\lim_{x\to\frac{\pi}{2}}\frac{-\not 2\eta\mu x}{\not 2}=-\eta\mu\frac{\pi}{2}=\boxed{-1}$$

18.1 5)

$$\lim_{x\to-\infty}\frac{2^{-x}}{x-5}\overset{\frac{+\infty}{-\infty}}{\underset{DLH}{=}}\lim_{x\to-\infty}\frac{(2^{-x})'}{(x-5)'}=\lim_{x\to-\infty}\frac{-2^{-x}\ln 2}{1}=-\infty$$

18.1 6)

$$\boxed{\lim_{x\to 0}\frac{\eta\mu^3 x}{3x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to 0}\frac{(\eta\mu^3 x)'}{(3x)'}=\lim_{x\to 0}\frac{3\eta\mu^2 x\cdot\sigma\upsilon\nu x}{3}=\frac{3\cdot 0\cdot 1}{3}=\boxed{0}$$

18.1 7)

$$\boxed{\lim_{x\to 0}\frac{\eta\mu x-x^2}{x^3-x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to 0}\frac{(\eta\mu x-x^2)'}{(x^3-x)'}=\lim_{x\to 0}\frac{\sigma\upsilon\nu x-2x}{3x^2-1}=\frac{1-2\cdot 0}{3\cdot 0^2-1}=\boxed{-1}$$

18.1 8)

$$\boxed{\lim_{x\to -2}\frac{e^{-2}-e^x}{x+2}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to -2}\frac{(e^{-2}-e^x)'}{(x+2)'}=\lim_{x\to -2}\frac{-e^x}{1}=\boxed{-e^{-2}}$$

18.1 9)

$$\boxed{\lim_{x\to1}\frac{2^{x-1}-3^{x-1}}{x-1}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to1}\frac{\left(2^{x-1}-3^{x-1}\right)'}{(x-1)'}=\lim_{x\to1}\frac{2^{x-1}\ln2-3^{x-1}\ln3}{1}=2^0\ln2-3^0\ln3=$$

$$=\ln2-\ln3=\boxed{\ln\frac{2}{3}}$$

18.1 10)

$$\boxed{\lim_{x\to0}\frac{1-\sigma\upsilon\nu x}{\eta\mu x+x\sigma\upsilon\nu x}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to0}\frac{(1-\sigma\upsilon\nu x)'}{(\eta\mu x+x\sigma\upsilon\nu x)'}=\lim_{x\to0}\frac{\eta\mu x}{\sigma\upsilon\nu x+\sigma\upsilon\nu x-x\eta\mu x}=\frac{0}{1+1-0}=\boxed{0}$$

18.1 11)

$$\lim_{x\to+\infty}\frac{x^2+x}{\ln x}\ \underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}}\ \lim_{x\to+\infty}\frac{\left(x^2+x\right)'}{(\ln x)'}=\lim_{x\to+\infty}\frac{2x+1}{\frac{1}{x}}=\lim_{x\to+\infty}\left(2x^2+x\right)=\lim_{x\to+\infty}2x^2=+\infty$$

18.1 12)

$$\boxed{\lim_{x\to0}\frac{\ln(\sigma\upsilon\nu 3x)}{3x}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to0}\frac{\left(\ln(\sigma\upsilon\nu 3x)\right)'}{(3x)'}=\lim_{x\to0}\frac{\frac{1}{\sigma\upsilon\nu 3x}\cdot(-\eta\mu 3x)\cdot\cancel{3}}{\cancel{3}}=\frac{-\eta\mu(3\cdot0)}{\sigma\upsilon\nu(3\cdot0)}=\boxed{0}$$

18.1 13)

$$\boxed{\lim_{x\to0}\frac{e^x+\eta\mu x-1}{\ln(x+1)}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to0}\frac{\left(e^x+\eta\mu x-1\right)'}{\left(\ln(x+1)\right)'}=\lim_{x\to0}\frac{e^x+\sigma\upsilon\nu x}{\frac{1}{x+1}}=\frac{e^0+\sigma\upsilon\nu 0}{\frac{1}{0+1}}=\boxed{2}$$

18.1 14)

$$\boxed{\lim_{x\to1^+}\frac{\ln x}{\sqrt{x-1}}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to1^+}\frac{(\ln x)'}{\left(\sqrt{x-1}\right)'}=\lim_{x\to1^+}\frac{\frac{1}{x}}{\frac{1}{2\sqrt{x-1}}}=\lim_{x\to1^+}\frac{2\sqrt{x-1}}{x}=\frac{2\sqrt{1-1}}{1}=\boxed{0}$$

18.1 15)

$$\boxed{\lim_{x\to1}\frac{\ln x+x-1}{x^2-e^{x-1}+1}}=\frac{\ln1+1-1}{1^2-e^{1-1}+1}=\frac{0}{1}=\boxed{0}$$

18.1 16)

$$\boxed{\lim_{x\to+\infty}\frac{e^{2x}}{3x+\ln4x}}\ \underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}}\ \lim_{x\to+\infty}\frac{\left(e^{2x}\right)'}{(3x+\ln4x)'}=\lim_{x\to+\infty}\frac{2e^{2x}}{3+\frac{1}{\cancel{4}x}\cdot\cancel{4}}=\frac{2\cdot(+\infty)}{3+\frac{1}{+\infty}}=\frac{+\infty}{3}=\boxed{+\infty}$$

18.1 17)

$$\boxed{\lim_{x\to0}\frac{e^x-1}{\ln(x+1)}}\ \underset{DLH}{\overset{\frac{0}{0}}{=}}\ \lim_{x\to0}\frac{\left(e^x-1\right)'}{\left[\ln(x+1)\right]'}=\lim_{x\to0}\frac{e^x}{\frac{1}{x+1}}=\lim_{x\to0}e^x(x+1)=e^0(0+1)=\boxed{1}$$

18.1 18)

$$\boxed{\lim_{x\to 0}\frac{\ln(x+1)}{x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{[\ln(x+1)]'}{(x)'} = \lim_{x\to 0}\frac{\frac{1}{x+1}}{1} = \lim_{x\to 0}\frac{1}{x+1} = \frac{1}{0+1} = \boxed{1}$$

18.1 19)

$$\boxed{\lim_{x\to 0}\frac{\ln(3x+1)}{3x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{[\ln(3x+1)]'}{(3x)'} = \lim_{x\to 0}\frac{\frac{1}{3x+1}\cancel{3}}{\cancel{3}} = \lim_{x\to 0}\frac{1}{3x+1} = \frac{1}{3\cdot 0+1} = \boxed{1}$$

18.1 20)

$$\boxed{\lim_{x\to +\infty}\frac{\ln x}{\ln(x+2)}} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to +\infty}\frac{(\ln x)'}{[\ln(x+2)]'} = \lim_{x\to +\infty}\frac{\frac{1}{x}}{\frac{1}{x+2}} = \lim_{x\to +\infty}\frac{x+2}{x} = \lim_{x\to +\infty}\frac{x}{x} = 1$$

18.1 21)

$$\boxed{\lim_{x\to 0}\frac{1-\sigma\upsilon\nu x}{2\eta\mu^2 x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to 0}\frac{\cancel{\eta\mu x}}{4\,\cancel{\eta\mu x}\cdot\sigma\upsilon\nu x} = \frac{1}{4\cdot 1} = \boxed{\frac{1}{4}}$$