

17.9 1)

Είναι

$$y = 4x^2 - 5x + 1 \xRightarrow{x=3t-4} y(t) = 4(3t-4)^2 - 5(3t-4) + 1 \Rightarrow$$

$$\Rightarrow y'(t) = 24(3t-4) - 5 \Rightarrow y'(t) = 72t - 111$$

Οπότε

α) $y'(1) = 72 \cdot 1 - 111 \Rightarrow y'(1) = -39$

β) $y'(2) = 72 \cdot 2 - 111 \Rightarrow y'(2) = 33$

γ) Είναι $x = 3t - 4 \xRightarrow{x=-4} -4 = 3t - 4 \Rightarrow 3t = 0 \Rightarrow t = 0$

Οπότε

$$y'(0) = 72 \cdot 0 - 111 \Rightarrow y'(0) = -111$$

δ) Είναι $x = 3t - 4 \xRightarrow{x=5} 5 = 3t - 4 \Rightarrow 3t = 9 \Rightarrow t = 3$

Οπότε

$$y'(3) = 72 \cdot 3 - 111 \Rightarrow y'(3) = 105$$

17.9 2)

Είναι

$$y = x^2 + 2x - 1 \xRightarrow{x=3t-4} y(t) = (3t-4)^2 + 2(3t-4) - 1 \Rightarrow$$

$$\Rightarrow y(t) = 9t^2 - 24t + 16 + 6t - 8 - 1 \Rightarrow y(t) = 9t^2 - 18t + 7 \Rightarrow \boxed{y'(t) = 18t - 18}$$

Οπότε

α) $y'(1) = 18 \cdot 1 - 18 \Rightarrow \boxed{y'(1) = 0}$

β) $y'(4) = 18 \cdot 4 - 18 \Rightarrow \boxed{y'(4) = 54}$

γ) Είναι $x = 3t - 4 \xRightarrow{x=-4} -4 = 3t - 4 \Rightarrow 3t = 0 \Rightarrow t = 0$

Οπότε

$$y'(0) = 18 \cdot 0 - 18 \Rightarrow \boxed{y'(0) = -18}$$

δ) Είναι $x = 3t - 4 \xRightarrow{x=2} 2 = 3t - 4 \Rightarrow 3t = 6 \Rightarrow t = 2$

Οπότε

$$y'(2) = 18 \cdot 2 - 18 \Rightarrow \boxed{y'(2) = 18}$$

17.9 3)

Είναι

$$y = x^3 + x^2 - 3x + 9 \xRightarrow{x=2t-6} y(t) = (2t-6)^3 + (2t-6)^2 - 3(2t-6) + 9 \Rightarrow$$

$$\Rightarrow \boxed{y'(t) = 6(2t-6)^2 + 4(2t-6) - 6}$$

Οπότε

α) $y'(2) = 6(2 \cdot 2 - 6)^2 + 4(2 \cdot 2 - 6) - 6 \Rightarrow \boxed{y'(2) = 10}$

β) $y'(0) = 6(2 \cdot 0 - 6)^2 + 4(2 \cdot 0 - 6) - 6 \Rightarrow \boxed{y'(0) = 186}$

γ) Είναι $x = 2t - 6 \stackrel{x=-4}{\Rightarrow} -4 = 2t - 6 \Rightarrow 2t = 2 \Rightarrow t = 1$

Οπότε

$$y'(1) = 6(2 \cdot 1 - 6)^2 + 4(2 \cdot 1 - 6) - 6 \Rightarrow \boxed{y'(1) = 74}$$

δ) Είναι $x = 2t - 6 \stackrel{x=2}{\Rightarrow} 2 = 2t - 6 \Rightarrow 2t = 8 \Rightarrow t = 4$

Οπότε

$$y'(4) = 6(2 \cdot 4 - 6)^2 + 4(2 \cdot 4 - 6) - 6 \Rightarrow \boxed{y'(4) = 26}$$

17.9 4)

Είναι

$$y = 4x^2 - 5x + 1 \stackrel{x=t^2+t-1}{\Rightarrow} y(t) = 4(t^2 + t - 1)^2 - 5(t^2 + t - 1) + 1 \Rightarrow$$

$$\Rightarrow \boxed{y'(t) = 8(t^2 + t - 1)(2t + 1) - 5(2t + 1)}$$

Οπότε

α) $y'(3) = 8(3^2 + 3 - 1)(2 \cdot 3 + 1) - 5(2 \cdot 3 + 1) \Rightarrow \boxed{y'(3) = 581}$

β) Είναι $x = t^2 + t - 1 \stackrel{x=-1}{\Rightarrow} \cancel{1} = t^2 + t \cancel{1} \Rightarrow t^2 + t = 0 \Rightarrow t(t + 1) = 0 \Rightarrow$
 $\Rightarrow t = 0 \text{ ή } t = -1 \text{ απορίπτεται}$

Οπότε

$$y'(0) = 8(0^2 + 0 - 1)(2 \cdot 0 + 1) - 5(2 \cdot 0 + 1) \Rightarrow \boxed{y'(0) = -13}$$

γ) Είναι $x = t^2 + t - 1 \stackrel{x=5}{\Rightarrow} 5 = t^2 + t - 1 \Rightarrow t^2 + t - 6 = 0 \Rightarrow (t - 2)(t + 3) = 0 \Rightarrow$
 $\Rightarrow t = 2 \text{ ή } t = -3 \text{ απορίπτεται}$

Οπότε

$$y'(2) = 8(2^2 + 2 - 1)(2 \cdot 2 + 1) - 5(2 \cdot 2 + 1) \Rightarrow \boxed{y'(2) = 175}$$

δ) Είναι $x = t^2 + t - 1 \stackrel{x=1}{\Rightarrow} 1 = t^2 + t - 1 \Rightarrow t^2 + t - 2 = 0 \Rightarrow (t - 1)(t + 2) = 0 \Rightarrow$
 $\Rightarrow t = 1 \text{ ή } t = -2 \text{ απορίπτεται}$

Οπότε

$$y'(1) = 8(1^2 + 1 - 1)(2 \cdot 1 + 1) - 5(2 \cdot 1 + 1) \Rightarrow \boxed{y'(1) = 9}$$