

Επειδή η f είναι πολυώνυμο με ρίζες ρ_1, ρ_2, ρ_3 θα έχει παράγοντα το $x - \rho_1$, το $x - \rho_2$ και το $x - \rho_3$. Επομένως επειδή είναι πολυώνυμο τρίτου βαθμού θα είναι

$$f(x) = \alpha(x - \rho_1)(x - \rho_2)(x - \rho_3)$$

Παραγωγίζοντας έχουμε

$$\begin{aligned} f'(x) &= \alpha(x - \rho_1)'(x - \rho_2)(x - \rho_3) + \alpha(x - \rho_1)(x - \rho_2)'(x - \rho_3) + \alpha(x - \rho_1)(x - \rho_2)(x - \rho_3)' = \\ f'(x) &= \alpha(x - \rho_2)(x - \rho_3) + \alpha(x - \rho_1)(x - \rho_3) + \alpha(x - \rho_1)(x - \rho_2) \quad (1) \end{aligned}$$

Επομένως

$$\begin{aligned} (1) \quad \overset{\text{θέτουμε } x=\rho_1}{\Rightarrow} f'(\rho_1) &= \alpha(\rho_1 - \rho_2)(\rho_1 - \rho_3) + \alpha(\rho_1 - \rho_1)(\rho_1 - \rho_3) + \alpha(\rho_1 - \rho_1)(\rho_1 - \rho_2) \Rightarrow \\ \Rightarrow f'(\rho_1) &= \alpha(\rho_1 - \rho_2)(\rho_1 - \rho_3) \quad (2) \end{aligned}$$

$$\begin{aligned} (1) \quad \overset{\text{θέτουμε } x=\rho_2}{\Rightarrow} f'(\rho_2) &= \alpha(\rho_2 - \rho_2)(\rho_2 - \rho_3) + \alpha(\rho_2 - \rho_1)(\rho_2 - \rho_3) + \alpha(\rho_2 - \rho_1)(\rho_2 - \rho_2) \Rightarrow \\ \Rightarrow f'(\rho_2) &= \alpha(\rho_2 - \rho_1)(\rho_2 - \rho_3) \Rightarrow f'(\rho_1) = -\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3) \quad (3) \end{aligned}$$

$$\begin{aligned} (1) \quad \overset{\text{θέτουμε } x=\rho_3}{\Rightarrow} f'(\rho_3) &= \alpha(\rho_3 - \rho_2)(\rho_3 - \rho_3) + \alpha(\rho_3 - \rho_1)(\rho_3 - \rho_3) + \alpha(\rho_3 - \rho_1)(\rho_3 - \rho_2) \Rightarrow \\ \Rightarrow f'(\rho_3) &= \alpha(\rho_3 - \rho_1)(\rho_3 - \rho_2) \Rightarrow f'(\rho_1) = \alpha(\rho_1 - \rho_3)(\rho_2 - \rho_3) \quad (4) \end{aligned}$$

Έτσι

$$\begin{aligned} &\boxed{\frac{\rho_1}{f'(\rho_1)} + \frac{\rho_2}{f'(\rho_2)} + \frac{\rho_3}{f'(\rho_3)}} \overset{(2), (3), (4)}{=} \\ &= \frac{\rho_1}{\alpha(\rho_1 - \rho_2)(\rho_1 - \rho_3)} - \frac{\rho_2}{\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3)} + \frac{\rho_3}{\alpha(\rho_1 - \rho_3)(\rho_2 - \rho_3)} = \\ &= \frac{\rho_1(\rho_2 - \rho_3)}{\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3)(\rho_1 - \rho_3)} - \frac{\rho_2(\rho_1 - \rho_3)}{\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3)(\rho_1 - \rho_3)} + \frac{\rho_3(\rho_1 - \rho_2)}{\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3)(\rho_1 - \rho_3)} = \\ &= \frac{\cancel{\rho_1\rho_2} - \cancel{\rho_1\rho_3} - \cancel{\rho_2\rho_1} + \cancel{\rho_2\rho_3} + \cancel{\rho_3\rho_1} - \cancel{\rho_3\rho_2}}{\alpha(\rho_1 - \rho_2)(\rho_2 - \rho_3)(\rho_1 - \rho_3)} = \boxed{0} \end{aligned}$$