

Είναι

$$\begin{aligned} f'(x) &= \ln x + \cancel{x} \frac{1}{\cancel{x}} + \alpha \sigma\upsilon\nu(\ln x) - \alpha \cancel{x} \eta\mu(\ln x) \frac{1}{\cancel{x}} + \beta \eta\mu(\ln x) + \beta \cancel{x} \sigma\upsilon\nu(\ln x) \frac{1}{\cancel{x}} \\ &= \ln x + 1 + \alpha \sigma\upsilon\nu(\ln x) - \alpha \eta\mu(\ln x) + \beta \eta\mu(\ln x) + \beta \sigma\upsilon\nu(\ln x) = \\ &= \boxed{\ln x + 1 + (\alpha + \beta) \sigma\upsilon\nu(\ln x) + (\beta - \alpha) \eta\mu(\ln x)} \end{aligned}$$

και

$$\begin{aligned} f''(x) &= \frac{1}{x} - \frac{(\alpha + \beta) \eta\mu(\ln x)}{x} + \frac{(\beta - \alpha) \sigma\upsilon\nu(\ln x)}{x} = \\ &= \boxed{\frac{1 - (\alpha + \beta) \eta\mu(\ln x) + (\beta - \alpha) \sigma\upsilon\nu(\ln x)}{x}} \end{aligned}$$

Οπότε

$$\begin{aligned} x^2 f''(x) - x f'(x) + 2f(x) &= \\ x \cancel{x} \frac{1 - (\alpha + \beta) \eta\mu(\ln x) + (\beta - \alpha) \sigma\upsilon\nu(\ln x)}{\cancel{x}} - \\ - x [\ln x + 1 + (\alpha + \beta) \sigma\upsilon\nu(\ln x) + (\beta - \alpha) \eta\mu(\ln x)] + \\ + 2 [x \ln x + \alpha x \sigma\upsilon\nu(\ln x) + \beta x \eta\mu(\ln x)] &= \\ \cancel{x} - (\alpha + \beta) x \eta\mu(\ln x) + (\beta - \alpha) x \sigma\upsilon\nu(\ln x) - \cancel{x \ln x} - \cancel{x} - (\alpha + \beta) x \sigma\upsilon\nu(\ln x) - \\ - (\beta - \alpha) x \eta\mu(\ln x) + \cancel{2} x \ln x + 2\alpha x \sigma\upsilon\nu(\ln x) + 2\beta x \eta\mu(\ln x) &= \\ x \eta\mu(\ln x) [-\cancel{\alpha} - \beta - \beta + \cancel{\alpha} + \cancel{2\beta}] + x \sigma\upsilon\nu(\ln x) [\beta - \cancel{\alpha} - \cancel{\alpha} - \beta + \cancel{2\alpha}] + x \ln x &= \boxed{x \ln x} \end{aligned}$$