

15.52

$$h(x) = f(x)g(x) \xRightarrow{\text{παραγωγίζουμε}} h'(x) = f'(x)g(x) + f(x)g'(x) \xRightarrow{\text{παραγωγίζουμε}} ,$$

$$\Rightarrow h''(x) = f''(x)g(x) + f'(x)g'(x) + f'(x)g'(x) + f(x)g''(x) \xRightarrow{f'(x)g'(x)'=c}$$

$$\Rightarrow h''(x) = f''(x)g(x) + 2c + f(x)g''(x) \xRightarrow{\text{παραγωγίζουμε}}$$

$$\Rightarrow h'''(x) = f'''(x)g(x) + f''(x)g'(x) + f'(x)g''(x) + f(x)g'''(x) \Rightarrow$$

$$\begin{array}{l} f'(x)g'(x)=c \xRightarrow{\text{παραγωγίζουμε}} \\ \boxed{f''(x)g'(x)+f'(x)g''(x)=0} \\ \Rightarrow h'''(x) = f'''(x)g(x) + f(x)g'''(x) \end{array}$$