

Καταρχήν έχουμε

$$f(x^2) = f(x) \xRightarrow{\text{παραγωγίζουμε}} 2xf'(x^2) = f'(x) \xRightarrow{x=1} 2 \cdot 1 \cdot f'(1) = f'(1) \Rightarrow f'(1) = 0$$

Επομένως

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{xf(x) - 2}{x - 1} &\stackrel{\text{προσθαφαιρούμε } xf(1)}{=} \lim_{x \rightarrow 1} \frac{xf(x) - xf(1) + xf(1) - 2}{x - 1} = \\ &= \lim_{x \rightarrow 1} \frac{xf(x) - xf(1)}{x - 1} + \lim_{x \rightarrow 1} \frac{xf(1) - 2}{x - 1} \stackrel{f(1)=2}{=} \lim_{x \rightarrow 1} \frac{x[f(x) - f(1)]}{x - 1} + \lim_{x \rightarrow 1} \frac{2x - 2}{x - 1} = \\ &= \lim_{x \rightarrow 1} x \cdot \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} + \lim_{x \rightarrow 1} \frac{2(x-1)}{\cancel{x-1}} = 1 \cdot f'(1) + 2 \stackrel{f'(1)=0}{=} \boxed{2} \end{aligned}$$