

Έχουμε

$$f(x) \sin x = e^x g(x) - 3g(x) \sin x \quad \xRightarrow{\text{διαφορ. με } g(x) \sin x \neq 0}$$

$$\Rightarrow \frac{f(x) \cancel{\sin x}}{g(x) \cancel{\sin x}} = \frac{e^x \cancel{g(x)}}{\cancel{g(x)} \sin x} - \frac{3 \cancel{g(x)} \sin x}{\cancel{g(x)} \sin x} \Rightarrow \frac{f(x)}{g(x)} = \frac{e^x}{\sin x} - 3 \quad \xRightarrow{\text{παραγωγίζουμε}}$$

$$\Rightarrow \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)} = \frac{e^x \sin x + e^x \eta \mu x}{\sin^2 x} \quad \xRightarrow{x=0}$$

$$\Rightarrow \frac{f'(0) \cdot g(0) - f(0) \cdot g'(0)}{g^2(0)} = \frac{e^0 \sin 0 + e^0 \eta \mu 0}{\sin^2 0} \Rightarrow \frac{f'(0) \cdot g(0) - f(0) \cdot g'(0)}{g^2(0)} = 1 \Rightarrow$$

$$\Rightarrow \boxed{f'(0)g(0) - f(0)g'(0) = g^2(0)}$$