# Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

## 15.42 1)

Για κάθε $x \in f(A) = (-1,1)$ είναι

$$f\left(f^{-1}(x)\right) = x \overset{\text{παραγωγίζουμε}}{\Rightarrow} f'\left(f^{-1}(x)\right)\left[f^{-1}(x)\right]' = 1 \overset{f'(x)=\sqrt{1-f^2(x)}}{\Rightarrow}$$

$$\sqrt{1-f^2\left(f^{-1}(x)\right)}\left[f^{-1}(x)\right]' = 1 \overset{f^2\left(f^{-1}(x)\right)=\left[f\left(f^{-1}(x)\right)\right]^2=x^2}{\Rightarrow} \sqrt{1-x^2}\left[f^{-1}(x)\right]' = 1 \Rightarrow$$

$$\overset{x\in(-1,1)\Rightarrow\sqrt{1-x^2}\neq 0}{\Rightarrow} \boxed{\left[f^{-1}(x)\right]' = \frac{1}{\sqrt{1-x^2}}}$$

## 15.42 2)

Για κάθε $x \in \left(0, \frac{\pi}{2}\right)$ είναι

$$f^{-1}\left(f(x)\right) = x \overset{\text{παραγωγίζουμε}}{\Rightarrow} \left(f^{-1}\right)'\left(f(x)\right)\cdot f'(x) = 1 \overset{\text{θέτουμε } x=\frac{\pi}{4}}{\Rightarrow}$$

$$\Rightarrow \left(f^{-1}\right)'\left(f\left(\frac{\pi}{4}\right)\right)\cdot f'\left(\frac{\pi}{4}\right) = 1 \overset{\substack{f\left(\frac{\pi}{4}\right)=\frac{\pi}{6} \\ f'\left(\frac{\pi}{4}\right)=\ln^2\left(\eta\mu^2\frac{\pi}{4}\right)=\ln^2\left(\frac{\sqrt{2}}{2}\right)^2=\ln^2\frac{1}{2}=(-\ln 2)^2=\ln^2 2}}{\Rightarrow} \left(f^{-1}\right)'\left(\frac{\pi}{6}\right)\cdot \ln^2 2 = 1 \Rightarrow$$

$$\Rightarrow \left(f^{-1}\right)'\left(\frac{\pi}{6}\right) = \frac{1}{\ln^2 2}$$

## 15.42 3)

Για κάθε $x \in \left(e^{-\pi}, e^{\pi}\right)$ είναι

$$f^{-1}\left(f(x)\right) = x \overset{\text{παραγωγίζουμε}}{\Rightarrow} \left(f^{-1}\right)'\left(f(x)\right)\cdot f'(x) = 1 \overset{\text{θέτουμε } x=e^{\frac{\pi}{3}}}{\Rightarrow}$$

$$\Rightarrow \left(f^{-1}\right)'\left(f\left(e^{\frac{\pi}{3}}\right)\right)\cdot f'\left(e^{\frac{\pi}{3}}\right) = 1 \overset{\substack{f\left(e^{\frac{\pi}{3}}\right)=e^{\frac{\pi}{4}} \\ f'\left(e^{\frac{\pi}{3}}\right)=\eta\mu\left(\ln\frac{1}{\sqrt{e^{\frac{\pi}{3}}}}\right)=\eta\mu\left[\ln\left(e^{\frac{\pi}{3}}\right)^{-\frac{1}{2}}\right]=\eta\mu\left(\ln e^{-\frac{\pi}{6}}\right)=\eta\mu\left(-\frac{\pi}{6}\right)=-\frac{1}{2}}}{\Rightarrow}$$

$$\Rightarrow \left(f^{-1}\right)'\left(e^{\frac{\pi}{4}}\right)\cdot\left(-\frac{1}{2}\right) = 1 \Rightarrow \left(f^{-1}\right)'\left(e^{\frac{\pi}{4}}\right) = -2$$