Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.41 1)

Είναι

$$f^2(x)+g^2(x)-x^2=0 \overset{x=1}{\Rightarrow} f^2(1)+g^2(1)-1^2=0 \overset{f(1)=0}{\Rightarrow} g^2(1)=1 \overset{g(1)>0}{\Rightarrow} \boxed{g(1)=1}$$

Οπότε

$$f^2(x)+g^2(x)-x^2=0 \overset{\text{παραγωγίζουμε}}{\Rightarrow} 2f(x)\cdot f'(x)+2g(x)\cdot g'(x)-2x=0 \overset{x=1}{\Rightarrow}$$

$$\Rightarrow 2f(1)\cdot f'(1)+2g(1)\cdot g'(1)-2\cdot 1=0 \overset{\substack{f(1)=0\\ g(1)=1}}{\Rightarrow} 2g'(1)=2 \Rightarrow \boxed{g'(1)=1}$$

15.41 2)

$$f(x^2)=x^3 \overset{\text{παραγωγίζουμε}}{\Rightarrow} f'(x^2)\cdot 2x=3x^2 \quad (1)$$

Οπότε

$$(1) \overset{x=1}{\Rightarrow} f'(1^2)\cdot 2\cdot 1=3\cdot 1^2 \Rightarrow 2\cdot f'(1)=3 \Rightarrow \boxed{f'(1)=\frac{3}{2}}$$

Ακόμη

$$(1) \overset{x=\sqrt{2}}{\Rightarrow} f'\left(\sqrt{2}^2\right)\cdot 2\cdot\sqrt{2}=3\cdot\sqrt{2}^2 \Rightarrow 2\cdot\sqrt{2}\cdot f'(2)=6 \Rightarrow f'(2)=\frac{\cancel{6}\,3}{\cancel{2}\cdot\sqrt{2}} \Rightarrow \boxed{f'(2)=\frac{3\sqrt{2}}{2}}$$

15.41 3)

Είναι

$$f''(x)=f^2(x)\cdot f'(x) \overset{x=0}{\Rightarrow} f''(0)=f^2(0)\cdot f'(0) \Rightarrow f''(0)=(-1)^2\cdot 1 \Rightarrow \boxed{f''(0)=1}$$

Οπότε

$$f''(x)=f^2(x)\cdot f'(x) \overset{\text{παραγωγίζουμε}}{\Rightarrow} f'''(x)=2f(x)\cdot\left[f'(x)\right]^2+f^2(x)f''(x) \overset{x=0}{\Rightarrow}$$

$$\Rightarrow \boxed{f'''(0)}=2f(0)\cdot\left[f'(0)\right]^2+f^2(0)f''(0)=2\cdot(-1)\cdot 1^2+(-1)^2\cdot 1=-2+1=\boxed{-1}$$

15.41 4)

Είναι

$$f^3(x)-g^2(x)+x=0 \overset{x=0}{\Rightarrow} f^3(0)-g^2(0)+0=0 \overset{g(0)=1}{\Rightarrow} f^3(0)=1 \Rightarrow \boxed{f(0)=1}$$

Οπότε

$$f^3(x)-g^2(x)+x=0 \overset{\text{παραγωγίζουμε}}{\Rightarrow} 3f^2(x)\cdot f'(x)-2g(x)\cdot g'(x)+1=0 \overset{x=0}{\Rightarrow}$$

$$\Rightarrow 3f^2(0)\cdot f'(0)-2g(0)\cdot g'(0)+1=0 \overset{\substack{f(0)=1\\ g(0)=g'(0)=1}}{\Rightarrow} 3f'(0)-2+1=0 \Rightarrow \boxed{f'(0)=\frac{1}{3}}$$

15.41 5)

Είναι

$$\varphi(x)=\frac{f(x)}{g(x)}\overset{\text{παραγωγίζουμε}}{\Rightarrow}\varphi'(x)=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$$

Οπότε

$$\varphi'(\rho)=0\Rightarrow\frac{f'(\rho)\cdot g(\rho)-f(\rho)\cdot g'(\rho)}{g^2(\rho)}=0\Rightarrow f'(\rho)\cdot g(\rho)-f(\rho)\cdot g'(\rho)=0\Rightarrow$$

$$\Rightarrow f'(\rho)\cdot g(\rho)=f(\rho)\cdot g'(\rho)\overset{\substack{g(\rho)\neq0\\ g'(\rho)\neq0\text{ διότι αν }g'(\rho)=0\Rightarrow f'(\rho)=0\text{ άτοπο}}}{\Rightarrow}\frac{f'(\rho)}{g'(\rho)}=\frac{f(\rho)}{g(\rho)}\overset{\varphi(\rho)=\frac{f(\rho)}{g(\rho)}}{\Rightarrow}$$

$$\Rightarrow\boxed{\varphi(\rho)=\frac{f'(\rho)}{g'(\rho)}}$$

15.41 6)

Είναι

$$g(x)=(x^2+1)f'(x)\overset{\text{παραγωγίζουμε}}{\Rightarrow}g'(x)=2x\cdot f'(x)+(x^2+1)f''(x)$$

Οπότε

$$\boxed{\frac{g'(x)}{g(x)}-\frac{f''(x)}{f'(x)}}\overset{\substack{g(x)=(x^2+1)f'(x)\\ g'(x)=2x\cdot f'(x)+(x^2+1)f''(x)}}{=}\frac{2x\cdot f'(x)+(x^2+1)f''(x)}{(x^2+1)f'(x)}-\frac{f''(x)}{f'(x)}=$$

$$=\frac{2x\cdot \cancel{f'(x)}}{(x^2+1)\cancel{f'(x)}}+\frac{\cancel{(x^2+1)}f''(x)}{\cancel{(x^2+1)}f'(x)}-\frac{f''(x)}{f'(x)}=\frac{2x}{x^2+1}+\frac{f''(\cancel{x})}{\cancel{f'}(x)}-\frac{f''(\cancel{x})}{\cancel{f'}(x)}=\boxed{\frac{2x}{x^2+1}}$$

15.41 7)

Είναι

$$e^{f(\ln x)}=g(x^2)\overset{x=1}{\Rightarrow}e^{f(\ln 1)}=g(1^2)\Rightarrow\boxed{e^{f(0)}=g(1)}$$

Οπότε

$$e^{f(\ln x)}=g(x^2)\overset{\text{παραγωγίζουμε}}{\Rightarrow}e^{f(\ln x)}f'(\ln x)\cdot\frac{1}{x}=g'(x^2)\cdot 2x\overset{x=1}{\Rightarrow}$$

$$\Rightarrow e^{f(\ln 1)}f'(\ln 1)\cdot\frac{1}{1}=g'(1^2)\cdot2\cdot1\Rightarrow e^{f(0)}f'(0)=2g'(1)\overset{e^{f(0)}=g(1)}{\Rightarrow}g(1)f'(0)=2g'(1)\overset{g(1)=e^{f(0)}\neq0}{=}$$

$$\Rightarrow\boxed{\frac{f'(0)}{2}=\frac{g'(1)}{g(1)}}$$