

15.39 1)

Έστω $P(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$ (1), ένα πολυώνυμο τρίτου βαθμού. Τότε

$$P'(x) = 3\alpha x^2 + 2\beta x + \gamma \quad (2) \text{ και } P''(x) = 6\alpha x + 2\beta \quad (3)$$

Οπότε

$$P(0) = 1 \Rightarrow \alpha \cdot 0^3 + \beta \cdot 0^2 + \gamma \cdot 0 + \delta = 1 \Rightarrow \boxed{\delta = 1}$$

$$P'(0) = -3 \Rightarrow 3\alpha \cdot 0^2 + 2\beta \cdot 0 + \gamma = -3 \Rightarrow \boxed{\gamma = -3}$$

$$P''(0) = 2 \Rightarrow 6\alpha \cdot 0 + 2\beta = 2 \Rightarrow 2\beta = 2 \Rightarrow \boxed{\beta = 1}$$

$$P'(1) = 5 \Rightarrow 3\alpha \cdot 1^2 + 2\beta \cdot 1 + \gamma = 5 \stackrel{\beta=1, \gamma=-3}{\Rightarrow} 3\alpha + 2 - 3 = 5 \Rightarrow \boxed{\alpha = 2}$$

Οπότε τελικά $P(x) = 2x^3 + x^2 - 3x + 1$

15.39 2)

Έστω $P(x) = \alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \varepsilon$ (1), ένα πολυώνυμο τετάρτου βαθμού. Τότε

$$P'(x) = 4\alpha x^3 + 3\beta x^2 + 2\gamma x + \delta \quad (2) \text{ και } P''(x) = 12\alpha x^2 + 6\beta x + 2\gamma \quad (3)$$

Οπότε

$$P(0) = 0 \Rightarrow \alpha \cdot 0^4 + \beta \cdot 0^3 + \gamma \cdot 0^2 + \delta \cdot 0 + \varepsilon = 0 \Rightarrow \boxed{\varepsilon = 0}$$

$$P'(0) = 1 \Rightarrow 4\alpha \cdot 0^3 + 3\beta \cdot 0^2 + 2\gamma \cdot 0 + \delta = 1 \Rightarrow \boxed{\delta = 1}$$

$$P''(0) = -6 \Rightarrow 12\alpha \cdot 0^2 + 6\beta \cdot 0 + 2\gamma = -6 \Rightarrow 2\gamma = -6 \Rightarrow \boxed{\gamma = -3}$$

$$P(1) = 1 \Rightarrow \alpha \cdot 1^4 + \beta \cdot 1^3 + \gamma \cdot 1^2 + \delta \cdot 1 + \varepsilon = 1 \stackrel{\varepsilon=0, \delta=1, \gamma=-3}{\Rightarrow} \alpha + \beta - 3 + 1 = 1 \Rightarrow \boxed{\alpha + \beta = 3} \quad (4)$$

$$P'(1) = 5 \Rightarrow 4\alpha \cdot 1^3 + 3\beta \cdot 1^2 + 2\gamma \cdot 1 + \delta = 5 \stackrel{\delta=1, \gamma=-3}{\Rightarrow} 4\alpha + 3\beta - 6 + 1 = 5 \Rightarrow \boxed{4\alpha + 3\beta = 10} \quad (5)$$

Λύνοντας το σύστημα των (1) και (2) βρίσκουμε $\boxed{\alpha = 1}$ και $\boxed{\beta = 2}$

Οπότε τελικά $P(x) = x^4 + 2x^3 - 3x^2 + x$