Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.37 1)

$$f(x)=\eta\mu(\alpha x+\beta)\Rightarrow f'(x)=\sigma\upsilon\nu(\alpha x+\beta)(\alpha x+\beta)'\overset{(\alpha x+\beta)'=\alpha}{\Rightarrow} f'(x)=\alpha\sigma\upsilon\nu(\alpha x+\beta)$$

και άρα

$$f''(x)=-\alpha\eta\mu(\alpha x+\beta)(\alpha x+\beta)'\overset{(\alpha x+\beta)'=\alpha}{\Rightarrow} f''(x)=-\alpha^2\eta\mu(\alpha x+\beta)$$

Οπότε

$$\boxed{f''(x)+\alpha^2 f(x)}\overset{\substack{f''(x)=-\alpha^2\eta\mu(\alpha x+\beta)\\ f(x)=\eta\mu(\alpha x+\beta)}}{=}-\alpha^2\eta\mu(\alpha x+\beta)+\alpha^2\eta\mu(\alpha x+\beta)=\boxed{0}$$

15.37 2)

Είναι $f'(x)=\dfrac{x\sigma\upsilon\nu x-\eta\mu x}{x^2}$. Άρα

$$\boxed{xf'(x)+f(x)}=\cancel{x}\frac{x\sigma\upsilon\nu x-\eta\mu x}{x^{\cancel{2}}}+\frac{\eta\mu x}{x}=\frac{x\sigma\upsilon\nu x-\cancel{\eta\mu x}+\cancel{\eta\mu x}}{x}=\frac{\cancel{x}\sigma\upsilon\nu x}{\cancel{x}}=\boxed{\sigma\upsilon\nu x}$$

15.37 3)

Είναι $f'(x)=\dfrac{-2x\cdot 2x-(4-x^2)\cdot 2}{4x^2}=\dfrac{-4x^2-8+2x^2}{4x^2}=\dfrac{-2x^2-8}{4x^2}$. Άρα

$$\boxed{xf'(x)+f(x)+x}=\cancel{x}\frac{-2x^2-8}{4x^{\cancel{2}}}+\frac{4-x^2}{2x}+x=\frac{-2x^2-\cancel{8}+\cancel{8}-2x^2+4x^2}{4x}=\boxed{0}$$

15.37 4)

Είναι $f'(x)=\dfrac{1}{\dfrac{1}{\cancel{1+x}}}\cdot\left(-\dfrac{1}{(1+x)^{\cancel{2}}}\right)=-\dfrac{1}{1+x}$. Άρα

$$\left.\begin{aligned}&\boxed{xf'(x)+1}=x\cdot\left(-\frac{1}{1+x}\right)+1=\frac{-x}{1+x}+\frac{1+x}{1+x}=\frac{-\cancel{x}+1+\cancel{x}}{1+x}=\boxed{\frac{1}{1+x}}\\ &\boxed{e^{f(x)}}=e^{\ln\left(\frac{1}{1+x}\right)}=\boxed{\frac{1}{1+x}}\end{aligned}\right\}\Rightarrow xf'(x)+1=e^{f(x)}$$

15.37 5)

Είναι $\boxed{f(0)}=e^{-\frac{0}{\alpha}}\sigma\upsilon\nu\left(\dfrac{0}{\alpha}\right)=e^0\sigma\upsilon\nu 0=\boxed{1}$

Ακόμη $f'(x)=-e^{-\frac{x}{\alpha}}\dfrac{1}{\alpha}\sigma\upsilon\nu\left(\dfrac{x}{\alpha}\right)-e^{-\frac{x}{\alpha}}\eta\mu\left(\dfrac{x}{\alpha}\right)\cdot\dfrac{1}{\alpha}$. Άρα

$$\boxed{f'(0)}=-e^{-\frac{0}{\alpha}}\frac{1}{\alpha}\sigma\upsilon\nu\left(\frac{0}{\alpha}\right)-e^{-\frac{0}{\alpha}}\eta\mu\left(\frac{0}{\alpha}\right)\cdot\frac{1}{\alpha}=-e^0\frac{1}{\alpha}\sigma\upsilon\nu 0-e^0\eta\mu 0\cdot\frac{1}{\alpha}=\boxed{-\frac{1}{\alpha}}$$

Επομένως

$$\boxed{f(0)+\alpha f'(0)}=1+\cancel{\alpha}\left(-\frac{1}{\cancel{\alpha}}\right)=1-1=\boxed{0}$$

15.37 6)

Είναι $f'(x)=2\alpha x-\frac{\beta}{x^2}\Rightarrow f''(x)=2\alpha+\frac{\beta}{x^{\not 4\,3}}2\not x=2\alpha+\frac{2\beta}{x^3}$. Άρα

$$\boxed{x^2f''(x)}=x^2\cdot\left(2\alpha+\frac{2\beta}{x^3}\right)=2\alpha x^2+\not x^2\frac{2\beta}{x^{\not 3}}=2\alpha x^2+\frac{2\beta}{x}=2\left(\alpha x^2+\frac{\beta}{x}\right)=\boxed{2f(x)}$$

15.37 7)

Είναι

$$\boxed{f'(x)}=2\alpha e^{2x}+\beta e^{2x}+2\beta xe^{2x}$$

$$\boxed{f''(x)}=4\alpha e^{2x}+2\beta e^{2x}+2\beta e^{2x}+4\beta xe^{2x}=4\alpha e^{2x}+4\beta e^{2x}+4\beta xe^{2x}$$

Οπότε

$$\boxed{f''(x)-4f'(x)+4f(x)}=$$

$$=4\alpha e^{2x}+4\beta e^{2x}+4\beta xe^{2x}-4\left[2\alpha e^{2x}+\beta e^{2x}+2\beta xe^{2x}\right]+4\left[\alpha e^{2x}+\beta xe^{2x}\right]=$$

$$=\cancel{4\alpha e^{2x}}+\cancel{4\beta e^{2x}}+\cancel{4\beta xe^{2x}}-\cancel{8\alpha e^{2x}}-\cancel{4\beta e^{2x}}-\cancel{8\beta xe^{2x}}+\cancel{4\alpha e^{2x}}+\cancel{4\beta xe^{2x}}=\boxed{0}$$