Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.27 1)

α) Είναι $f(x)=x^{\sqrt{x}} \Rightarrow f(x)=e^{\ln x^{\sqrt{x}}} \Rightarrow f(x)=e^{\sqrt{x}\ln x}$

Επομένως $f'(x)=\left(e^{\sqrt{x}\ln x}\right)'=e^{\sqrt{x}\ln x}\left(\sqrt{x}\ln x\right)'=e^{\sqrt{x}\ln x}\left(\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right)$

β) Είναι $f(x)=(\eta\mu x)^{\varepsilon\varphi x} \Rightarrow f(x)=e^{\ln(\eta\mu x)^{\varepsilon\varphi x}}=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}$

Επομένως $f'(x)=\left(e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\right)'=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\varepsilon\varphi x\cdot\ln(\eta\mu x)\right]'=$

$$=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\frac{1}{\sigma\upsilon\nu^2 x}\cdot\ln(\eta\mu x)+\varepsilon\varphi x\cdot\frac{1}{\eta\mu x}\cdot\sigma\upsilon\nu x\right]$$

15.27 2)

$$f(x)=e^{\ln x^x}=e^{x\ln x}$$

$$\Rightarrow f'(x)=\left(e^{x\ln x}\right)'=e^{x\ln x}(x\ln x)'=e^{x\ln x}\left(\ln x+\cancel{x}\frac{1}{\cancel{x}}\right)=e^{x\ln x}(\ln x+1)$$

15.27 3)

$$f(x)=e^{\ln(1+\sqrt{x})^x}=e^{x\ln(1+\sqrt{x})}$$

$$\Rightarrow f'(x)=\left(e^{x\ln(1+\sqrt{x})}\right)'=e^{x\ln(1+\sqrt{x})}\cdot\left[x\ln(1+\sqrt{x})\right]'=$$

$$=e^{x\ln(1+\sqrt{x})}\cdot\left[\ln(1+\sqrt{x})+\frac{x}{1+\sqrt{x}}\frac{1}{2\sqrt{x}}\right]$$

15.27 4)

$$f(x)=e^{\ln(x^2-x+1)^{\eta\mu x}}=e^{\eta\mu x\ln(x^2-x+1)}$$

$$\Rightarrow f'(x)=\left(e^{\eta\mu x\ln(x^2-x+1)}\right)'=e^{\eta\mu x\ln(x^2-x+1)}\cdot\left[\eta\mu x\ln(x^2-x+1)\right]'=$$

$$e^{\eta\mu x\ln(x^2-x+1)}\cdot\left[\sigma\upsilon\nu x\ln(x^2-x+1)+\frac{\eta\mu x}{x^2-x+1}(2x-1)\right]$$

15.27 5)

$$\Rightarrow f(x)=e^{\ln(x^2+x+1)^{\ln x}}=e^{\ln x(x^2+x+1)}$$

$$\Rightarrow f'(x)=\left(e^{\ln x\cdot(x^2+x+1)}\right)'=e^{\ln x(x^2+x+1)}\left[\ln x\cdot(x^2+x+1)\right]'=$$

$$=e^{\ln x(x^2+x+1)}\left[\frac{x^2+x+1}{x}+(2x+1)\cdot\ln x\right]$$

15.27 6)

$$f(x)=e^{\ln(\eta\mu x)^x}=e^{x\ln(\eta\mu x)}$$

$$\Rightarrow f'(x)=\left(e^{x\ln(\eta\mu x)}\right)'=e^{x\ln(\eta\mu x)}\cdot\left[x\ln(\eta\mu x)\right]'=e^{x\ln(\eta\mu x)}\cdot\left[\ln(\eta\mu x)+\frac{x}{\eta\mu x}\sigma\upsilon\nu x\right]$$

15.27 7)

$$f(x)=\left(x^2+2\right)^{\sqrt{x}}\Rightarrow f(x)=e^{\ln\left(x^2+2\right)^{\sqrt{x}}}=e^{\sqrt{x}\ln\left(x^2+2\right)}$$

$$\Rightarrow f'(x)=\left(e^{\sqrt{x}\ln\left(x^2+2\right)}\right)'=e^{\sqrt{x}\ln\left(x^2+2\right)}\cdot\left[\sqrt{x}\ln\left(x^2+2\right)\right]'=$$

$$=e^{\sqrt{x}\ln\left(x^2+2\right)}\cdot\left[\frac{\ln\left(x^2+2\right)}{2\sqrt{x}}+\sqrt{x}\cdot\frac{1}{x^2+2}\cdot 2x\right]$$

15.27 8)

$$f(x)=(\ln x)^{\sigma\upsilon\nu x}\Rightarrow f(x)=e^{\ln(\ln x)^{\sigma\upsilon\nu x}}=e^{\sigma\upsilon\nu x\cdot\ln(\ln x)}$$

$$\Rightarrow f'(x)=\left(e^{\sigma\upsilon\nu x\cdot\ln(\ln x)}\right)'=e^{\sigma\upsilon\nu x\cdot\ln(\ln x)}\cdot\left[\sigma\upsilon\nu x\cdot\ln(\ln x)\right]'=$$

$$=e^{\sigma\upsilon\nu x\cdot\ln(\ln x)}\cdot\left[-\eta\mu x\cdot\ln(\ln x)+\sigma\upsilon\nu x\cdot\frac{1}{\ln x}\cdot\frac{1}{x}\right]=$$

$$=e^{\sigma\upsilon\nu x\cdot\ln(\ln x)}\cdot\left[-\eta\mu x\cdot\ln(\ln x)+\frac{\sigma\upsilon\nu x}{\ln x}\right]$$

15.27 9)

$$f(x)=\left(1+\frac{1}{x}\right)^x\Rightarrow f(x)=e^{\ln\left(1+\frac{1}{x}\right)^x}=e^{x\cdot\ln\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow f'(x)=\left(e^{x\cdot\ln\left(1+\frac{1}{x}\right)}\right)'=e^{x\cdot\ln\left(1+\frac{1}{x}\right)}\cdot\left[x\cdot\ln\left(1+\frac{1}{x}\right)\right]'=$$

$$=e^{x\cdot\ln\left(1+\frac{1}{x}\right)}\cdot\left[\ln\left(1+\frac{1}{x}\right)+\cancel{x}\cdot\frac{1}{1+\frac{1}{x}}\cdot\left(-\frac{1}{x^{\cancel{2}}}\right)\right]=e^{x\cdot\ln\left(1+\frac{1}{x}\right)}\cdot\left[\ln\left(1+\frac{1}{x}\right)+\frac{1}{\frac{x+1}{\cancel{x}}}\cdot\left(-\frac{1}{\cancel{x}}\right)\right]=$$

$$e^{x\cdot\ln\left(1+\frac{1}{x}\right)}\cdot\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]$$

15.27 10)

$$f(x)=\left(\frac{x+6}{3x}\right)^x\Rightarrow f(x)=e^{\ln\left(\frac{x+6}{3x}\right)^x}=e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}$$

$$\Rightarrow f'(x)=\left(e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\right)'=e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\cdot\left[x\cdot\ln\left(\frac{x+6}{3x}\right)\right]'=$$

$$=e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\cdot\left[\ln\left(\frac{x+6}{3x}\right)+x\cdot\frac{1}{\frac{x+6}{3x}}\cdot\frac{3x-3(x+6)}{(3x)^2}\right]=$$

$$=e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\cdot\left[\ln\left(\frac{x+6}{3x}\right)+x\cdot\frac{1}{\frac{x+6}{3\cancel{x}}}\cdot\frac{\cancel{3x}-\cancel{3x}-18}{9x^{\cancel{2}}}\right]=$$

$$e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\cdot\left[\ln\left(\frac{x+6}{3x}\right)+\cancel{x}\cdot\frac{1}{\frac{x+6}{3}}\cdot\frac{-\cancel{18}^{2}}{\cancel{9}\,\cancel{x}}\right]=e^{x\cdot\ln\left(\frac{x+6}{3x}\right)}\cdot\left[\ln\left(\frac{x+6}{3x}\right)-\frac{6}{x+6}\right]=$$

15.27 11)

$$f(x)=\left(5-2\sqrt{x}\right)^{3\sqrt{x}}\Rightarrow f(x)=e^{\ln\left(5-2\sqrt{x}\right)^{3\sqrt{x}}}=e^{3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)}$$

$$\Rightarrow f'(x)=\left(e^{3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)}\right)'=e^{3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)}\cdot\left[3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)\right]'=$$

$$=e^{3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)}\cdot\left[\frac{3\ln\left(5-2\sqrt{x}\right)}{2\sqrt{x}}-3\cancel{\sqrt{x}}\cdot\frac{1}{5-2\sqrt{x}}\cdot\frac{1\,\cancel{2}}{\cancel{2}\,\cancel{\sqrt{x}}}\right]=$$

$$=e^{3\sqrt{x}\cdot\ln\left(5-2\sqrt{x}\right)}\cdot\left[\frac{3\ln\left(5-2\sqrt{x}\right)}{2\sqrt{x}}-\frac{3}{5-2\sqrt{x}}\right]$$

15.27 12)

$$f(x)=\left(4+7\sqrt{x}\right)^{\sqrt{x}}\Rightarrow f(x)=e^{\ln\left(4+7\sqrt{x}\right)^{\sqrt{x}}}=e^{\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)}$$

$$\Rightarrow f'(x)=\left(e^{\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)}\right)'=e^{\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)}\cdot\left[\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)\right]'=$$

$$=e^{\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)}\cdot\left[\frac{1}{2\sqrt{x}}\ln\left(4+7\sqrt{x}\right)+\cancel{\sqrt{x}}\cdot\frac{1}{4+7\sqrt{x}}\cdot\frac{7}{2\cancel{\sqrt{x}}}\right]$$

$$=e^{\sqrt{x}\cdot\ln\left(4+7\sqrt{x}\right)}\cdot\left[\frac{\ln\left(4+7\sqrt{x}\right)}{2\sqrt{x}}+\frac{7}{2\left(4+7\sqrt{x}\right)}\right]$$

15.27 13)

$$f(x)=(\eta\mu x)^{\varepsilon\varphi x}\Rightarrow f(x)=e^{\ln(\eta\mu x)^{\varepsilon\varphi x}}=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}$$

$$\Rightarrow f'(x)=\left(e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\right)'=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\varepsilon\varphi x\cdot\ln(\eta\mu x)\right]'=$$

$$=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\frac{1}{\sigma\upsilon\nu^2 x}\cdot\ln(\eta\mu x)+\varepsilon\varphi x\cdot\frac{1}{\eta\mu x}\cdot\sigma\upsilon\nu x\right]=$$

$$=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\frac{\ln(\eta\mu x)}{\sigma\upsilon\nu^2 x}+\frac{\cancel{\eta\mu x}}{\cancel{\sigma\upsilon\nu x}}\cdot\frac{1}{\cancel{\eta\mu x}}\cdot\cancel{\sigma\upsilon\nu x}\right]=e^{\varepsilon\varphi x\cdot\ln(\eta\mu x)}\cdot\left[\frac{\ln(\eta\mu x)}{\sigma\upsilon\nu^2 x}+1\right]$$

15.27 14)

$$f(x)=(x-3)^{5^x}\Rightarrow f(x)=e^{\ln(x-3)^{5^x}}=e^{5^x\cdot\ln(x-3)}$$

$$\Rightarrow f'(x)=\left(e^{5^x\cdot\ln(x-3)}\right)'=e^{5^x\cdot\ln(x-3)}\cdot\left[5^x\cdot\ln(x-3)\right]'=$$

$$=e^{5^x\cdot\ln(x-3)}\cdot\left[5^x\ln5\cdot\ln(x-3)+\frac{5^x}{x-3}\right]$$

15.27 15)

$$f(x)=3x^{x^3}\Rightarrow f(x)=e^{\ln3x^{x^3}}=e^{x^3\cdot\ln3x}$$

$$\Rightarrow f'(x)=\left(e^{x^3\cdot\ln3x}\right)'=e^{x^3\cdot\ln3x}\cdot\left(x^3\cdot\ln3x\right)'=$$

$$=e^{x^3\cdot\ln3x}\cdot\left(3x^2\cdot\ln3x+x^{\not{3}^2}\cdot\frac{1}{\not{3}\not{x}}\cdot\not{3}\right)=e^{x^3\cdot\ln3x}\cdot\left(3x^2\cdot\ln3x+x^2\right)$$

15.27 16)

$$f(x)=x^{x^e}\Rightarrow f(x)=e^{\ln x^{x^e}}=e^{x^e\cdot\ln x}$$

$$\Rightarrow f'(x)=\left(e^{x^e\cdot\ln x}\right)'=e^{x^e\cdot\ln x}\cdot\left(x^e\cdot\ln x\right)'=e^{x^e\cdot\ln x}\cdot\left(ex^{e-1}\cdot\ln x+x^e\cdot\frac{1}{e}\right)=$$

$$=e^{x^e\cdot\ln x}\cdot\left(ex^{e-1}\cdot\ln x+\frac{x^e}{e}\right)$$

15.27 17)

$$f(x)=e^{\ln(\eta\mu2x)^{\sigma\upsilon\nu x}}=e^{\sigma\upsilon\nu x\cdot\ln(\eta\mu2x)}$$

$$\Rightarrow f'(x)=\left(e^{\sigma\upsilon\nu x\cdot\ln(\eta\mu2x)}\right)'=e^{\sigma\upsilon\nu x\cdot\ln(\eta\mu2x)}\cdot\left[\sigma\upsilon\nu x\cdot\ln(\eta\mu2x)\right]'=$$

$$=e^{\sigma\upsilon\nu x\cdot\ln(\eta\mu2x)}\cdot\left[-\eta\mu x\cdot\ln(\eta\mu2x)+\sigma\upsilon\nu x\cdot\frac{1}{\eta\mu2x}\cdot2\sigma\upsilon\nu2x\right]$$

15.27 18)

$$f(x)=(\sigma\upsilon\nu2x)^{\sigma\varphi x}\Rightarrow f(x)=e^{\ln(\sigma\upsilon\nu2x)^{\sigma\varphi x}}=e^{\sigma\varphi x\cdot\ln(\sigma\upsilon\nu2x)}$$

$$\Rightarrow f'(x)=\left(e^{\sigma\varphi x\cdot\ln(\sigma\upsilon\nu2x)}\right)'=e^{\sigma\varphi x\cdot\ln(\sigma\upsilon\nu2x)}\cdot\left[\sigma\varphi x\cdot\ln(\sigma\upsilon\nu2x)\right]'=$$

$$=e^{\sigma\varphi x\cdot\ln(\sigma\upsilon\nu2x)}\cdot\left[-\frac{1}{\eta\mu^2x}\cdot\ln(\sigma\upsilon\nu2x)-\sigma\varphi x\cdot\frac{1}{\sigma\upsilon\nu2x}2\eta\mu2x\right]$$

15.27 19)

$$f'(x)=\frac{1}{\eta\mu x^x}\sigma\upsilon\nu x^x\cdot\left(x^x\right)'\overset{\left(x^x\right)'=\left(e^{\ln x^x}\right)'=\left(e^{x\ln x}\right)'=e^{x\ln x}(x\ln x)'=e^{x\ln x}\left(\ln x+\not{x}\frac{1}{\not{x}}\right)=e^{x\ln x}(\ln x+1)}{=}$$

$$=\frac{1}{\eta\mu x^x}\sigma\upsilon\nu x^x\cdot e^{x\ln x}(\ln x+1)$$

15.27 20)

$$f'(x)=\frac{1}{\sigma\upsilon\nu^2x^x}\left(x^x\right)'\overset{\left(x^x\right)'=\left(e^{\ln x^x}\right)'=\left(e^{x\ln x}\right)'=e^{x\ln x}(x\ln x)'=e^{x\ln x}\left(\ln x+\not{x}\frac{1}{\not{x}}\right)=e^{x\ln x}(\ln x+1)}{=}$$

$$=\frac{1}{\sigma\upsilon\nu^2x^x}e^{x\ln x}(\ln x+1)$$