Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.24 1)

α) $\left(xe^{-\frac{1}{x}}\right)'=(x)'e^{-\frac{1}{x}}+x\left(e^{-\frac{1}{x}}\right)'=1\cdot e^{-\frac{1}{x}}+xe^{-\frac{1}{x}}\left(-\frac{1}{x}\right)'=e^{-\frac{1}{x}}+\cancel{x}e^{-\frac{1}{x}}\frac{1}{x^{\cancel{2}}}$

β) $\left[\ln\left(\frac{2^x-1}{x}\right)\right]'=\frac{1}{\frac{2^x-1}{x}}\left(\frac{2^x-1}{x}\right)'=\frac{\cancel{x}}{2^x-1}\cdot\frac{2^x\ln2\cdot x-2^x+1}{x^{\cancel{2}}}=\frac{2^x\ln2\cdot x-2^x+1}{x\left(2^x-1\right)}$

15.24 2)

$\left(5^{x+1}+\ln\sqrt{x}\right)'=\left(5^{x+1}\right)'+\left(\ln\sqrt{x}\right)'=5^{x+1}\ln5+\frac{1}{\sqrt{x}}\frac{1}{2\sqrt{x}}=5^{x+1}\ln5+\frac{1}{2x}$

15.24 3)

$\left(3^{2x-1}-5^{4x-7}\right)'=\left(3^{2x-1}\right)'-\left(5^{4x-7}\right)'=3^{2x-1}\ln3\cdot(2x-1)'+5^{4x-7}\ln5\cdot(4x-7)'=$

$=2\cdot3^{2x-1}\ln3+4\cdot5^{4x-7}\ln5$

15.24 4)

$\left(\ln^2x-\frac{4}{\varepsilon\varphi x}\right)'=\left(\ln^2x\right)'-\left(\frac{4}{\varepsilon\varphi x}\right)'=2\ln x\cdot(\ln x)'-4\left(-\frac{1}{\varepsilon\varphi^2x}\right)\cdot(\varepsilon\varphi x)'=$

$=2\ln x\cdot\frac{1}{x}-4\left(-\frac{1}{\varepsilon\varphi^2x}\right)\cdot\frac{1}{\sigma\upsilon\nu^2x}=\frac{2\ln x}{x}+\frac{4}{\frac{\eta\mu^2x}{\cancel{\sigma\upsilon\nu^2x}}}\cdot\frac{1}{\cancel{\sigma\upsilon\nu^2x}}=\frac{2\ln x}{x}+\frac{4}{\eta\mu^2x}$

15.24 5)

$\left(2e^{-5x}\sigma\upsilon\nu4x\right)'=\left(2e^{-5x}\right)'\sigma\upsilon\nu4x+2e^{-5x}(\sigma\upsilon\nu4x)'=$

$=\left(2e^{-5x}\right)(-5x)'\sigma\upsilon\nu4x+2e^{-5x}(-\eta\mu4x)\cdot(4x)'=$

$=\left(2e^{-5x}\right)(-5)\sigma\upsilon\nu4x-2e^{-5x}\eta\mu4x\cdot4=-10e^{-5x}\sigma\upsilon\nu4x-8e^{-5x}\eta\mu4x$

15.24 6)

$\left(\sigma\upsilon\nu^2x\eta\mu3x\right)'=\left(\sigma\upsilon\nu^2x\right)'\eta\mu3x+\sigma\upsilon\nu^2x(\eta\mu3x)'=$

$=2\sigma\upsilon\nu x(\sigma\upsilon\nu x)'\eta\mu3x+\sigma\upsilon\nu^2x(\sigma\upsilon\nu3x)(3x)'=$

$=2\sigma\upsilon\nu x(-\eta\mu x)\eta\mu3x+\sigma\upsilon\nu^2x\cdot\sigma\upsilon\nu3x\cdot3=-2\sigma\upsilon\nu x\cdot\eta\mu x\cdot\eta\mu3x+3\sigma\upsilon\nu^2x\cdot\sigma\upsilon\nu3x$

15.24 7)

$$\left[\frac{(x+3)^3}{\sqrt{x^2+4}}\right]'=\frac{\left[(x+3)^3\right]'\sqrt{x^2+4}-(x+3)^3\left(\sqrt{x^2+4}\right)'}{\left(\sqrt{x^2+4}\right)^2}=$$

$$=\frac{3(x+3)^2(x+3)'\sqrt{x^2+4}-(x+3)^3\frac{1}{2\sqrt{x^2+4}}\left(x^2+4\right)'}{\left(\sqrt{x^2+4}\right)^2}=$$

$$=\frac{3(x+3)^2\cdot 1\cdot\sqrt{x^2+4}-(x+3)^3\frac{1}{\cancel{2}\sqrt{x^2+4}}\cancel{2}x}{x^2+4}=\frac{3(x+3)^2\sqrt{x^2+4}-\frac{x(x+3)^3}{\sqrt{x^2+4}}}{x^2+4}$$

15.24 8)

$$\left[\left(\frac{8x+3}{x-5}\right)^3\right]'=3\left(\frac{8x+3}{x-5}\right)^2\cdot\left(\frac{8x+3}{x-5}\right)'=$$

$$=3\left(\frac{8x+3}{x-5}\right)^2\cdot\frac{8\cdot(x-5)-(8x+3)\cdot 1}{(x-5)^2}=3\left(\frac{8x+3}{x-5}\right)^2\cdot\frac{\cancel{8x}-40-\cancel{8x}-3}{(x-5)^2}=$$

$$=3\frac{(8x+3)^2}{(x-5)^2}\cdot\frac{-43}{(x-5)^2}=\frac{-129(8x+3)^2}{(x-5)^4}$$

15.24 9)

$$\left[5e^x\ln(x^2+2)\right]'=(5e^x)'\ln(x^2+2)+5e^x\left[\ln(x^2+2)\right]'=$$

$$=5e^x\ln(x^2+2)+5e^x\frac{1}{x^2+2}(x^2+2)'=5e^x\ln(x^2+2)+5e^x\frac{1}{x^2+2}2x=$$

$$=5e^x\ln(x^2+2)+\frac{10xe^x}{x^2+2}$$

15.24 10)

$$\left[(4x^3+1)\sqrt{x^2+8}\right]'=(4x^3+1)'\sqrt{x^2+8}+(4x^3+1)\left(\sqrt{x^2+8}\right)'=$$

$$=12x^2\sqrt{x^2+8}+(4x^3+1)\frac{1}{2\sqrt{x^2+8}}(x^2+8)'=$$

$$=12x^2\sqrt{x^2+8}+(4x^3+1)\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+8}}=12x^2\sqrt{x^2+8}+\frac{x(4x^3+1)}{\sqrt{x^2+8}}$$

15.24 11)

$$\left[\frac{1}{2}(e^x-e^{-x})\right]'=\frac{1}{2}(e^x-e^{-x})'=\frac{1}{2}\left[(e^x)'-(e^{-x})'\right]=\frac{1}{2}\left[e^x-e^{-x}(-x)'\right]=$$

$$=\frac{1}{2}\left[e^x-e^{-x}(-1)\right]=\frac{1}{2}(e^x+e^{-x})$$

15.24 12)

$$\left(\frac{1}{3}ημ^3x-\frac{1}{2}συν^2x\right)'=\left(\frac{1}{3}ημ^3x\right)'-\left(\frac{1}{2}συν^2x\right)'=$$

$$=\frac{1}{\cancel{3}}\cancel{3}ημ^2x(ημx)'-\frac{1}{\cancel{2}}\cancel{2}συνx(συνx)'=ημ^2x\cdot συνx-συνx(-ημx)=$$

$$=ημ^2x\cdot συνx+συνxημx$$

15.24 13)

$$\left[\sqrt{\frac{xημx}{1-συνx}}\right]'=\frac{1}{2\sqrt{\frac{xημx}{1-συνx}}}\left(\frac{xημx}{1-συνx}\right)'=$$

$$=\frac{1}{2\sqrt{\frac{x\eta\mu x}{1-\sigma\upsilon\nu x}}}\cdot\frac{(x\eta\mu x)'(1-\sigma\upsilon\nu x)-x\eta\mu x(1-\sigma\upsilon\nu x)'}{(1-\sigma\upsilon\nu x)^2}=$$

$$=\frac{1}{2\sqrt{\frac{x\eta\mu x}{1-\sigma\upsilon\nu x}}}\cdot\frac{\left[(x)'(\eta\mu x)+(x)(\eta\mu x)'\right](1-\sigma\upsilon\nu x)-x\eta\mu x\left[-(-\eta\mu x)\right]}{(1-\sigma\upsilon\nu x)^2}=$$

$$=\frac{1}{2\sqrt{\frac{x\eta\mu x}{1-\sigma\upsilon\nu x}}}\cdot\frac{(\eta\mu x+x\sigma\upsilon\nu x)(1-\sigma\upsilon\nu x)-x\eta\mu^2 x}{(1-\sigma\upsilon\nu x)^2}$$

15.24 14)

$$\left[\left(2x^2+5\right)\ln\left(2x^2+5\right)\right]'=\left(2x^2+5\right)'\ln\left(2x^2+5\right)+\left(2x^2+5\right)\left[\ln\left(2x^2+5\right)\right]'=$$

$$=4x\ln\left(2x^2+5\right)+\cancel{\left(2x^2+5\right)}\frac{1}{\cancel{2x^2+5}}\left(2x^2+5\right)'=4x\ln\left(2x^2+5\right)+4x$$

15.24 15)

$$\left(\frac{\sqrt{x+4}-\sqrt{x-4}}{\sqrt{x+4}+\sqrt{x-4}}\right)'=$$

$$=\frac{\left(\sqrt{x+4}-\sqrt{x-4}\right)'\left(\sqrt{x+4}+\sqrt{x-4}\right)-\left(\sqrt{x+4}-\sqrt{x-4}\right)\left(\sqrt{x+4}+\sqrt{x-4}\right)'}{\left(\sqrt{x+4}-\sqrt{x-4}\right)^2}$$

$$\left(\sqrt{x+4}\right)'=\frac{1}{2\sqrt{x+4}}(x+4)'=\frac{1}{2\sqrt{x+4}}\cdot1=\frac{1}{2\sqrt{x+4}}$$

$$\left(\sqrt{x-4}\right)'=\frac{1}{2\sqrt{x-4}}(x-4)'=\frac{1}{2\sqrt{x-4}}\cdot1=\frac{1}{2\sqrt{x-4}}$$

$$=$$

$$\frac{\left(\frac{1}{2\sqrt{x+4}}-\frac{1}{2\sqrt{x-4}}\right)\left(\sqrt{x+4}+\sqrt{x-4}\right)-\left(\frac{1}{2\sqrt{x+4}}+\frac{1}{2\sqrt{x-4}}\right)\left(\sqrt{x+4}+\sqrt{x-4}\right)'}{\left(\sqrt{x+4}-\sqrt{x-4}\right)^2}$$

15.24 16)

$$\left[\left(\frac{2x-1}{x+3}\right)^5\right]'=5\left(\frac{2x-1}{x+3}\right)^4\left(\frac{2x-1}{x+3}\right)'=$$

$$5\left(\frac{2x-1}{x+3}\right)^4\frac{(2x-1)'(x+3)-(2x-1)(x+3)'}{(x+3)^2}=5\left(\frac{2x-1}{x+3}\right)^4\frac{2(x+3)-(2x-1)\cdot1}{(x+3)^2}=$$

$$=5\left(\frac{2x-1}{x+3}\right)^4\frac{\cancel{2x}+6-\cancel{2x}+1}{(x+3)^2}=5\frac{(2x-1)^4}{(x+3)^4}\frac{7}{(x+3)^2}=\frac{35(2x-1)^4}{(x+3)^6}$$

15.24 17)

$$\left(\frac{1-\varepsilon\varphi^2x}{1+\varepsilon\varphi^2x}\right)'=\frac{\left(1-\varepsilon\varphi^2x\right)'\left(1+\varepsilon\varphi^2x\right)-\left(1-\varepsilon\varphi^2x\right)\left(1+\varepsilon\varphi^2x\right)'}{\left(1+\varepsilon\varphi^2x\right)^2}=$$

$$=\frac{-2\varepsilon\varphi x(\varepsilon\varphi x)'\left(1+\varepsilon\varphi^2x\right)-\left(1-\varepsilon\varphi^2x\right)2\varepsilon\varphi x(\varepsilon\varphi x)'}{\left(1+\varepsilon\varphi^2x\right)^2}=$$

$$=\frac{-2\text{εφ}x\frac{1}{\text{συν}^2x}\left(1+\text{εφ}^2x\right)-\left(1-\text{εφ}^2x\right)2\text{εφ}x\frac{1}{\text{συν}^2x}}{\left(1+\text{εφ}^2x\right)^2}=\frac{\frac{2\text{εφ}x}{\text{συν}^2x}\left[-\left(1+\text{εφ}^2x\right)-\left(1-\text{εφ}^2x\right)\right]}{\left(1+\text{εφ}^2x\right)^2}=$$

$$=\frac{\frac{2\text{εφ}x}{\text{συν}^2x}\left(-1-\cancel{\text{εφ}^2x}-1+\cancel{\text{εφ}^2x}\right)}{\left(1+\text{εφ}^2x\right)^2}=\frac{\frac{2\text{εφ}x}{\text{συν}^2x}(-2)}{\left(1+\text{εφ}^2x\right)^2}=\frac{-4\text{εφ}x}{\text{συν}^2x\left(1+\text{εφ}^2x\right)^2}$$

15.24 18)

$$\left(x^2\text{σφ}\frac{1}{x}\right)'=\left(x^2\right)'\text{σφ}\frac{1}{x}+x^2\left(\text{σφ}\frac{1}{x}\right)'=2x\text{σφ}\frac{1}{x}+x^2\frac{-1}{\text{ημ}^2\frac{1}{x}}\left(\frac{1}{x}\right)'=$$

$$=2x\text{σφ}\frac{1}{x}+\cancel{x^2}\frac{-1}{\text{ημ}^2\frac{1}{x}}\left(-\frac{1}{\cancel{x^2}}\right)=2x\text{σφ}\frac{1}{x}+\frac{1}{\text{ημ}^2\frac{1}{x}}$$

15.24 19)

$$\left(7^{\frac{4x-3}{x+1}}\right)'=7^{\frac{4x-3}{x+1}}\ln 7\cdot\frac{(4x-3)'(x+1)-(4x-3)(x+1)'}{(x+1)^2}=$$

$$=7^{\frac{4x-3}{x+1}}\ln 7\cdot\frac{4(x+1)-(4x-3)\cdot 1}{(x+1)^2}=7^{\frac{4x-3}{x+1}}\ln 7\cdot\frac{\cancel{4x}+4-\cancel{4x}+3}{(x+1)^2}=7^{\frac{4x-3}{x+1}}\ln 7\cdot\frac{7}{(x+1)^2}$$

15.24 20)

$$\left(\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2+1}}\right)'=\frac{\left(x-\sqrt{x^2-1}\right)'\left(x+\sqrt{x^2+1}\right)-\left(x-\sqrt{x^2-1}\right)\left(x+\sqrt{x^2+1}\right)'}{\left(x+\sqrt{x^2+1}\right)^2}=$$

$$=\frac{\left(1-\frac{1}{2\sqrt{x^2-1}}\left(x^2-1\right)'\right)\left(x+\sqrt{x^2+1}\right)-\left(x-\sqrt{x^2-1}\right)\left(1-\frac{1}{2\sqrt{x^2+1}}\left(x^2+1\right)'\right)}{\left(x+\sqrt{x^2+1}\right)^2}=$$

$$=\frac{\left(1-\frac{2x}{2\sqrt{x^2-1}}\right)\left(x+\sqrt{x^2+1}\right)-\left(x-\sqrt{x^2-1}\right)\left(1-\frac{2x}{2\sqrt{x^2+1}}\right)}{\left(x+\sqrt{x^2+1}\right)^2}=$$